UNIVERSIDAD DE INVESTIGACIÓN DE TECNOLOGÍA EXPERIMENTAL YACHAY

**Escuela de Ciencias Físicas y Nanotecnología**

# TÍTULO: Numerical Study of the Higgs Inflationary Model

Trabajo de integración curricular presentado como requisito para la obtención del título de Físico

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# Resumen

El concepto de inflación cosmológica surgió como una solución propuesta para abordar varias deficiencias notables dentro del marco de la teoría del Big Bang Caliente[1], especialmente en lo que respecta a problemas como la planitud y los horizontes. En el centro de este enfoque está la identificación de un potencial que impulse esta época de expansión rápida. En este estudio, se elige el modelo de Higgs como punto focal de investigación. Este análisis emplea dos metodologías distintas: la aproximación de slow-roll y cálculos numéricos. A través de estos enfoques, se emprende una exploración exhaustiva de la dinámica e implicaciones del modelo de Higgs en el contexto de la cosmología inflacionaria. Los resultados de esta investigación arrojan ideas cruciales sobre el comportamiento del universo durante la época inflacionaria. Específicamente, se presta atención a características observacionales clave, incluido el índice espectral escalar representado como $n_{\rm S}$, la razón tensor-escalar representada por $r$, y el espectro de potencia escalar representado como $P_{\rm S}$. Estos resultados se comparan rigurosamente con datos observacionales, como los proporcionados por los resultados de Planck en 2018. La culminación de este trabajo revela evidencia convincente que sugiere que el modelo de Higgs tiene considerables ventajas cuando se compara con los hallazgos de la misión Planck. Además, intrigantemente, se observa que existe un espacio de parámetros dentro del cual se manifiesta una convergencia entre las predicciones numéricas y los conjuntos de datos observacionales. Esta convergencia subraya la viabilidad potencial y la relevancia del modelo de Higgs dentro del contexto más amplio de la investigación cosmológica contemporánea.



**Palabras Clave:** Cosmología, Inflación, slow-roll, Espectro de potencia, Potencial Higgs.

# Abstract

The concept of cosmological inflation emerged as a proposed solution to address several notable shortcomings within the framework of the Hot Big Bang theory [1], particularly concerning issues such as flatness and horizon problems. Central to this approach is identifying a potential that drives this epoch of rapid expansion. In this study, the Higgs model is chosen as the focal point for investigation. This analysis employs two distinct methodologies: the slow-roll approximation and numerical calculations. These approaches comprehensively explore the dynamics and implications of the Higgs model in the context of inflationary cosmology. The outcomes of this investigation yield crucial insights into the Universe's behavior during the inflationary epoch. Specifically, attention is directed towards key observational features, including the scalar spectral index denoted as $n_S$, the tensor-scalar ratio represented by $r$, and the scalar power spectrum denoted as $P_S$. These results are rigorously compared against observational data, such as those provided by Planck's results in 2018. The culmination of this work reveals compelling evidence suggesting that the Higgs model holds considerably favorable when juxtaposed with the findings of the Planck mission. Moreover, intriguingly, it is observed that a parameter space exists within which a convergence between numerical predictions and observational data sets is manifest. This convergence underscores the potential viability and relevance of the Higgs model within the broader context of contemporary cosmological inquiry.



**Keywords:** Cosmology, Inflation, slow-roll, Power Spectrum, Higgs potential.

# Contents

# List of Figures

# List of Tables

# Chapter 1

# Introduction

Understanding the cosmic phenomena shaping the Universe has been a continuous challenge for the scientific community and curiosity for humankind. In this context, cosmology stands as a crucial field of study aiming to unveil the secrets of the evolution, structure, and fundamental nature of the cosmos, And passing through the Einstein field equations, to which Friedmann provided solutions taking into account the temporal components[3]. Recent advancements in particle physics have led to new theoretical perspectives, and among these, the Higgs potential emerges as an essential component.

The Higgs potential, renowned for its role in the spontaneous breaking of electroweak symmetry, has proven to be a fundamental concept in particle physics. However, its application in cosmology has been a continually expanding area of research. This thesis delves into the fascinating domain of numerical modeling, exploring how the Higgs potential can be integrated to comprehend cosmic phenomena on a broader scale.

The primary objective of this work is to develop and apply numerical models incorporating the Higgs potential within the framework of cosmology. As we enter the era of precision in astronomical observation, having theoretical models that align with existing observations and predict unexplored phenomena is crucial. This study aims to fill this gap by proposing an innovative approach that integrates the Higgs potential into numerical simulations of cosmic events.

Through this research, we anticipate shedding light on key aspects of cosmic evolution, from forming large-scale structures to understanding dark energy. This work will contribute to advancing knowledge in cosmology and offer a unique perspective on the role of the Higgs potential in the configuration and dynamics of the Universe. Ultimately, this study represents a significant step toward the convergence of particle physics and cosmology, merging theory and observation to unveil the deepest mysteries of the cosmos. It is also worth noting that values such as $\hbar$, $M_{Pl}$ and $c$ are all set to 1, making it easier to compute complex equations.

## 1.1 Big Bang Theory

The Big Bang theory emerged approximately a hundred years ago with the discovery that all galaxies in the Universe are not static. This discovery was theoretically predicted by Lemaitre and Friedman and subsequently confirmed through Edwin Hubble's experiment. However, although the Big Bang theory successfully describes various Universe properties, it still lacks a complete explanation. One crucial aspect it fails to address fully is the Universe's origin. To delve deeper into this question, cosmologists turn to the framework of inflationary cosmology. This theory, which builds upon the foundations of the Big Bang theory, focuses on specific epochs in the universe's early history, particularly the rapid expansion phase known as cosmic inflation. By incorporating inflationary dynamics, cosmologists aim to elucidate the mechanisms driving the universe's initial expansion and the emergence of its fundamental properties. Thus, while the Big Bang theory provides a broad understanding of cosmic evolution, inflationary cosmology offers a more detailed account of the universe's earliest moments.

Before delving into this theory, we start with cosmological principles established from observations. Two crucial principles assert that the Universe is homogeneous and isotropic. A homogeneous Universe refers to the idea that, at large scales, the distribution of matter in the Universe is uniform in all directions. In other words, the density of matter is the same at all points in space.

When we say the Universe is isotropic, its physical properties are the same in all directions. In other words, no preferred directions exist for observing the Universe in space. On a large scale, the Universe's characteristics, such as matter density, cosmic expansion, and other fundamental properties, are uniform regardless of the direction you observe.

This large-scale homogeneity is a key concept in cosmological models, such as the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, describing an expanding, homogeneous, and isotropic Universe. In this context, isotropic means that the physical properties of the Universe are the same in all directions. Therefore, isotropy is an important concept in cosmological models, just as large-scale homogeneity is a key concept for using the Friedmann-Lemaître-Robertson-Walker (FLRW) metric[4 5 6], describing an expanding, homogeneous, and isotropic Universe within the framework of the Big Bang theory,

$$ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right], \tag{1.1}$$

Where $g_{\mu\nu}$ represents the components of a metric tensor in the theory of general relativity. This metric tensor describes the geometry of spacetime in a given coordinate system. The Greek indices $\mu$ and $\nu$ take values from 0 to 3. This equation is expressed in polar coordinates where $t, r, \theta$, and $\phi$ are the comoving coordinates. These mean coordinates expand with the universe, simplifying the description of cosmic phenomena. Additionally, we have the curvature parameter $K$, which can take on values according to the following table:

| K value | Curve[7] | Geometry[8] |
|---|---|---|
| -1 | Negative | Hyper spherical, |
| 0 | Flat | Euclidean |
| 1 | Positive | Spherical |

Table 1.1: Curvature and geometry according to $K$ values.

We are discussing expansion; we must delve into space-time deformation using the scale factor $a$ associated with the Universe's expansion.

However, to understand better, we need to return to more fundamental equations. Therefore, we solve the Einstein field equations. To simplify mathematical expressions, we will use natural units such as the speed of light, the gravitational constant, and the reduced Planck constant, which are set to 1.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{1.2}$$

Now, we need to determine which values we can nullify in our equation. First of all, we know that the Universe is expanding, so we can dispense with the cosmological constant $\Lambda$ obtaining:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{1.3}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \tag{1.4}$$

*"Space-time tells matter how to move; matter tells space-time how to curve"* John Wheeler.

Where G is the Newtonian constant of universal gravitation, $g_{\mu\nu}$ is the metric (FRW), $T_{\mu\nu}$ is the energy-momentum covariant tensor, $G_{\mu\nu}$ belongs to the covariant Einstein tensor, $R_{\mu\nu}$ is the covariant Ricci Tensor, and $R$ is the Ricci scalar.

Not only does the cosmological constant disappear from the equation, but now we will also look for values of the Einstein tensor that are different from zero:

$$G_0^0 = 3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2}\right], \tag{1.5}$$

$$G_j^i = \left[2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2}\right]\delta_{ij}, \tag{1.6}$$

Where $g_j^i = \delta_{ij}$, and we follow the same path to vanish the zero values of the Ricci tensor,

$$R_{00} = -3\frac{\ddot{a}}{a}, \tag{1.7}$$

$$R_{ij} = \left[\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{K}{a^2}\right]g_{ij}. \tag{1.8}$$

With the non-zero values of the Ricci tensor $R_{\mu\nu}$, the metric $g_{\mu\nu}$, and the Ricci scalar $R$, we can express the energy-momentum tensor $T_{\mu\nu}$in the following way. We must consider the Universe a perfect fluid because it simplifies the mathematical description of its dynamics and helps explain the origin of large-scale structures. meaning it has an energy density $\rho$ and the pressure $p$[9]. In that way, we can express the tensor as:

$$T_\mu^\nu = \begin{pmatrix} -\rho(t) & 0 & 0 & 0 \\ 0 & p(t) & 0 & 0 \\ 0 & 0 & p(t) & 0 \\ 0 & 0 & 0 & p(t) \end{pmatrix}. \tag{1.9}$$

Considering the density properties of energy, $\rho(t)$, and pressure, $p(t)$, and treating it as a perfect fluid, we need to derive the equation of state. For this, we replace Einstein's equation (1.3) and the Einstein tensor different from zero (1.5 and 1.6) into the diagonal components of the Energy-Momentum tensor matrix (1.9), so that we obtain:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho, \tag{1.10}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = -8\pi G p, \tag{1.11}$$

With the Friedman equations in hand (1.10) and (1.11), we can obtain a relation for cosmic acceleration[10] as follows:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right). \tag{1.12}$$

If, in addition, we derive the Friedman equation (1.10), we can obtain the following relationship, which will be useful for us:

$$\frac{8\pi G}{3}\dot{\rho} = \left(\frac{\dot{a}}{a}\right)\left[2\frac{\ddot{a}}{a} - 2\left(\frac{\dot{a}}{a}\right)^2 - 2\frac{K}{a^2}\right], \tag{1.13}$$

so, we can obtain the following equation.

$$\dot{\rho} + 3\frac{\dot{a}}{a}(p + \rho) = 0. \tag{1.14}$$

Almost finished, we can see that we obtained an equation similar to the energy conservation equation. To do this, we need to take the definition of $H$[11].

$$H = \frac{\dot{a}}{a}. \tag{1.15}$$

According to Andrew Liddle[11], the definition of $H$ refers to the Hubble constant, which represents the rate of expansion of the universe with respect to time. This constant is fundamental in describing the dynamics of the universe. With this definition of $H$[11], we can take Friedman's equation (1.10) and the energy conservation equation (1.14) and express them in the following way:

$$\dot{\rho} + 3H(p + \rho) = 0, \tag{1.16}$$

$$(H)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho. \tag{1.17}$$

These two equations are known as the **energy conservation equation**(1.16) and the **Friedman equation**(1.17), respectively.
As we start from the idea that the Universe is a perfect fluid, our equation of state is

$$p(t) = w\rho(t). \tag{1.18}$$

And if we now substitute this equation(1.18) into the energy conservation equation(1.16), we obtain:

$$\frac{d\rho}{\rho} = -3(1 + w)\frac{da}{a}. \tag{1.19}$$

With the equation of state at this point, we need to know in which epoch of the Universe[12 13] we are going to apply it because depending on that, different results will unfold. To do this, we rely on Table (1.2) with various values of $w$ depending on the epoch under consideration.

| ***Epoch*** | $w$ | *a(t)* |
|---|---|---|
| ***Radiation-dominated*** | 1/3 | $t^{1/2}$ |
| ***Matter-Dominated*** | 0 | $t^{2/3}$ |
| ***Dark Energy-dominated*** | -1 | *Sitter solution* |

Table 1.2: Values of a and $w$ according to the epoch.

The cosmological model we depict with the Big Bang theory provides a remarkable description of the Universe on cosmic scales with a fairly acceptable degree of approximation. However, as we observe the phenomena in our vast Universe, we encounter certain incongruities that pique the curiosity of the scientific community. Among them are the flatness problem[14] and the horizon problem[14], and we will delve into these complexities in detail below. Far from overshadowing the achievements of our cosmic understanding, these questions serve as stimuli for the quest for deeper answers and a clearer comprehension of the Universe.

## 1.2 Problems

The Universe has always been a source of mystery and awe for us. Since the Big Bang, scientists have been trying to understand the Universe's earliest moments and have encountered several enigmas. Two of these fundamental problems are the flatness problem and the horizon problem.

The flatness problem is about the uniformity of the density of the observable Universe[15], which is surprisingly consistent, defying expectations of significant fluctuations. On the other hand, the horizon problem arises due to the limited ability of distant regions of the Universe to send signals or information to one another. This is due to the vast distances between galaxies and the limitations of the speed of light. Despite this lack of direct contact, the Universe continues to evolve and expand uniformly.

Scientists are constantly working to understand the nature of these cosmic phenomena and unlock the Universe's secrets. These challenges represent significant mysteries that drive the search for a unified theory capable of explaining the fundamental nature of the cosmos in its earliest moments, according to the theory of relativity, which would only have had enough time to interact and thermally equilibrate from the Big Bang to the present day.

### 1.2.1 Horizon

The horizon problem refers to the limits of the causality at recombination epoch [16], an apparent contradiction between the observed large-scale homogeneity of the Universe and the Big Bang theory. According to the standard Big Bang model, the Universe originated from an extremely dense and hot state. However, when we observe different regions of the sky today, we find that they exhibit very uniform temperatures, even though some have not had enough time to communicate with each other due to the finite speed of light.

This apparent homogeneity in temperatures across distant regions of the Universe poses a dilemma because these regions haven't had enough time since the Big Bang to exchange thermal information. According to the theory of relativity, the maximum speed at which information can propagate is the speed of light. If two regions haven't had time to exchange information, they are expected to exhibit different properties, such as temperature.

The horizon problem suggests that we should observe more significant fluctuations in the Universe's properties on larger scales than we observe. The solution to this problem involves the introduction of a mechanism, such as cosmic inflation, that allows regions of the Universe that haven't had time to communicate with each other to acquire similar properties. Cosmic inflation posits a rapid period of Universe expansion in its early stages, resolving the horizon problem by allowing distant regions to become more homogeneous before the accelerated expansion.

To understand this problem through calculations, we can refer to the angular diameter of the horizon at the time of recombination.

To better understand the implications of the statement above, let us compute the angular diameter of the horizon at recombination as viewed by an observer today. Essentially, this is the size of the horizon at the last scattering surface [17]. divided by the current angular distance to the last scattering surface. In other words, it can be represented as:

$$\Delta\Omega = \frac{d_{\mathrm{H}}(t_{\mathrm{ls}})}{d_{\mathrm{A}}(t_{\mathrm{ls}})}. \tag{1.20}$$

At the epoch of the last scattering, $d_{\mathrm{H}}(t_{\mathrm{ls}})$ gives us the size of the horizon. In contrast, $d_{\mathrm{A}}(t_{\mathrm{ls}})$ represents the present angular distance to that moment.

To achieve this, a specific coordinate system is considered, where the origin is located on Earth. In this context, a photon may be emitted from spatial comoving coordinates $(r_{\mathrm{em}}, \theta_{\mathrm{em}}, \psi_{\mathrm{em}})$ but is described using cosmic time, this refers to time measured on the scale of the universe as a whole $t_{em}$. It is known that the geodesic equation provides a solution for calculating the path taken by a photon. This calculation allows for the coordinates $\theta$ and $\psi$ to remain constant while the coordinate $r$ becomes a function of time denoted as $r(t)$. In other words, the photon's path can be characterized solely by the changing value of $r$ over time.

$$r(t) = \left( r_{\text{em}} - \int_{t_{\text{em}}}^{t} \frac{d\tau}{a(\tau)} \right). \tag{1.21}$$

Next, it is crucial to derive an equation representing the proper distance between the location of the emitted photon at time $t$ and the origin. This parameter is commonly called the horizon size at the specific time $t = t_{\text{rec}}$, denoting a particular reception time.

$$d_{\text{p}}(t) = a(t) \left[ r_{\text{em}} - \int_{t_{\text{em}}}^{t} \frac{d\tau}{a(\tau)} \right]. \tag{1.22}$$

When the value of $d_{\text{p}}(t_{\text{rec}})$ is zero, we obtain the comoving coordinate of emission:

$$r_{\text{em}} = \int_{0}^{t_{\text{rec}}} \frac{d\tau}{a(\tau)}. \tag{1.23}$$

With this information, it becomes possible to express the distance to the horizon at time $t = t_{\text{rec}}$. It is important to note that this can be considered at any given time, such as $t = t_{\text{ls}}$. Consequently, this provides the initial term for our $\Delta\Omega$ expression.

$$d_{\text{H}}(t_{\text{rec}}) = a(t_{\text{rec}}) r_{\text{em}}. \tag{1.24}$$

The second aspect focuses on the distance originating from a point where a photon is emitted at time $t = t_{\text{em}}$ and reaches the Earth in the present moment.

$$r_{\text{em}} = \int_{t_{\text{em}}}^{t_0} \frac{d\tau}{a(\tau)}. \tag{1.25}$$

With this value we can calculate $d_{\text{A}}$:

$$d_{\text{A}} = a(t_{\text{em}}) r_{\text{em}}. \tag{1.26}$$

This expression is evaluated once more when $t = t_{\text{ls}}$:

$$d_{\text{A}}(t_{\text{ls}}) = a(t_{\text{ls}}) r_{\text{em}}. \tag{1.27}$$

In addition:

$$\Delta\Omega = \int_{0}^{t_{\text{ls}}} \frac{d\tau}{a(\tau)} \left[ \int_{t_{\text{ls}}}^{t_0} \frac{d\tau}{a(\tau)} \right]^{-1}. \tag{1.28}$$

where the terms $a(t_{\text{ls}})$ are canceled out, and the expression relates the term $r_{\text{em}}$.

The subsequent step involves introducing an expression for the quantity $a(t)$ and incorporating it into the equation, mentioned earlier. Martin[17] adopts the assumption of a Universe characterized by radiation dominance before the recombination epoch, transitioning to a matter-dominated state afterward. Within this framework, the radiation-dominated era is interrupted in the interval $t_{\rm i} < t < t_{\rm end}$, marked by the occurrence of inflation. During this phase, the influence of an unknown fluid emerges, adhering to a fixed equation of state parameterized by $\omega_x$.

A piece-wise function is employed to model this, ensuring continuity and smoothness in its first derivative.

$$a(t) = \begin{cases} a_{\rm i}\sqrt{(2H_{\rm i}t)} & , \; 0 \le t < t_{\rm i} \\ a_{\rm i}\left[\frac{3}{2}(1+\omega_x)H_i(t-t_{\rm i})+1\right]^{\frac{2}{3+3\omega_x)}} & , \; t_i \le t < t_{\rm end} \\ a_{\rm end}\sqrt{2H_{\rm end}(t-t_{\rm end}+1)} & , \; t_{\rm end} \le t < t_{\rm eq} \\ a_{\rm eq}\left[\frac{3H_{\rm eq}}{2}(t-t_{\rm eq})+1\right]^{\frac{2}{3}} & , \; t_{\rm eq} \le t < t_0 \end{cases} . \tag{1.29}$$

The subscript *eq* used in the parameters indicates the period of equilibrium, characterized by the point in time when the densities of radiation and matter reach a comparable level. This occurs during the intermediary phase between the epochs dominated by radiation and matter.

Subsequently, by substituting into the expressions as mentioned above, specifically Equation (1.24) and Equation (1.27), the resulting values are obtained.

$$d_{\rm A}(t_{\rm ls}) = a_{\rm ls}\int_{t_{\rm ls}}^{t_0}\frac{d\tau}{a(\tau)} = a_{\rm ls}\frac{2}{a_0H_0}\left[1-\left(\frac{a_{\rm ls}}{a_0}\right)^{\frac{1}{2}}\right], \tag{1.30}$$

$$d_{\rm H}(t_{\rm ls}) = a_{\rm ls}\int_0^{t_{\rm ls}}\frac{d\tau}{a(\tau)}, \tag{1.31}$$

$$= a_{\rm ls}\frac{1}{a_0H_0}\left(\frac{a_{\rm ls}}{a_0}\right)^{\frac{1}{2}}\left\{1+\frac{1-3\omega_x}{1+3\omega_x}\frac{a_{\rm end}}{a_{\rm ls}}\left[1-\left(\frac{a_{\rm i}}{a_{\rm end}}\right)^{\frac{(1+3\omega_x)}{2}}\right]\right\}. \tag{1.32}$$

The final expression, which is obtained by evaluating Equation (1.30) and Equation (1.32), into Equation (1.20).

$$\Delta\Omega = \frac{1}{2}\left[1-(1+z_{\rm ls})^{-\frac{1}{2}}\right]^{-1}(1+z_{\rm ls})^{-\frac{1}{2}}\left\{1+\frac{1-\omega_x}{1+3\omega_x}\frac{1+z_{\rm ls}}{1+z_{\rm end}}\left[1-e^{-N(1+3\omega_x)/2}\right]\right\}. \tag{1.33}$$

Here, $N \equiv \ln(a_{\rm end}/a_{\rm ini})$ symbolizes the number of e-foldings transpiring during inflation, where e-folding corresponds to a doubling in the scale factor of the universe, which essentially means that the size of the universe doubles during each e-folding. It's called "e-folding" because it's based on the mathematical constant "e," where "e" is approximately equal to 2.718. This concept will be explored more extensively in subsequent sections. In this particular instance, we consider the scenario where $N = 0$, signifying the absence of inflation. Consequently, the expression can be approximated as follows:

$$\Delta\Omega \approx 0.5(1 + z_{\text{ls}})^{-\frac{1}{2}} \approx 0.85^\circ. \tag{1.34}$$

The computation shows that the angular diameter of causally connected space during that period is smaller than $1^\circ$ [17]. Consequently, the sky would exhibit numerous patches with this angular diameter, each possessing distinct properties, diminishing the likelihood of homogeneity among them. This circumstance, coupled with the disparity between this expectation and observational evidence, gives rise to what is known as the horizon paradox problem[8]. Subsequent sections will delve into the discussion of how inflation addresses and resolves this paradox.

### 1.2.2 Flatness

The problem of flatness is closely related to situations of fine-tuning in cosmology, revealing an intricate connection between the geometry and the density of the Universe. This connection leads to a remarkably flat Universe in its current configuration, which, given the expected variations in density over cosmic time, proves to be a highly improbable situation[16]. To better understand this phenomenon, it is essential to begin by defining the density parameter, denoted as $\Omega$. This parameter quantifies the relationship between the current density of the Universe and its critical density, which determines the global geometry of the cosmos. In a flat Universe, $\Omega$ would equal 1. However, the problem of flatness arises from the need to explain why $\Omega$ is so close to 1 at present, despite the expectation that this value would vary considerably over cosmic time without specific fine-tuning adjustments in the Universe's initial conditions.

$$\Omega \equiv \frac{\rho}{\rho_{\text{cri}}}. \tag{1.35}$$

In this context, $\rho$ denotes the energy density, while $\rho_{\text{cri}}$ signifies its critical threshold, indicating the value at which the Universe would exhibit perfect flatness[17]. This is expressed as:

$$\rho_{\text{cri}}(t) = \frac{3H^2}{8\pi G}. \tag{1.36}$$

The current measurement of this parameter falls within the range $0.995 < \Omega_0 < 1.005$ [18], indicating that our Universe is extremely close to being flat. With this definition in mind, we will employ the Friedmann equation, which will be elaborated upon in the subsequent chapter, expressed as:

$$|\Omega_{\text{tot}}(t) - 1| = \frac{|k|}{a^2H^2}. \tag{1.37}$$

In the equation above, $\Omega_{\text{tot}}$ encompasses the aggregate of all types of matter present in the Universe. Additionally, the equation allows for the determination of the relationship between the curvature term $k$ and the energy density term $\Omega_{\text{tot}}$, as well as their temporal evolution[11].

If the total density parameter of the Universe, denoted as $\Omega_{\text{tot}}$, is equal to 1, the curvature term, represented by $k$, must remain at 0. This means that a flat Universe will remain flat. Otherwise, the density parameter will change over time[9]. Upon closer examination, it becomes clear that the relationship between the term $a^2H^2$ evolves depending on the dominant substance in the Universe.

$$\begin{aligned} a^2H^2 &\propto t^{-1} \quad \text{radiation;} \\ a^2H^2 &\propto t^{-2/3} \quad \text{matter;} \end{aligned} \tag{1.38}$$

Utilizing the connection provided by the Friedmann equation, Eq. (2.21) can be transformed into:

$$\begin{aligned} |\Omega_{\text{tot}} - 1| &\propto t^{1} \quad \text{radiation;} \\ |\Omega_{\text{tot}} - 1| &\propto t^{2/3} \quad \text{matter;} \end{aligned} \tag{1.39}$$

As evident, the deviation between the density parameter $\Omega_{\text{tot}}$ and unity is a function that amplifies as the Universe ages. This implies that a state of exact flatness is inherently unstable[11]. If, at any juncture, the density parameter strays significantly from 1, it will continue to diverge further. Should this parameter exceed 1, it would escalate until gravitational collapse ensues, leading to a closed Universe model. Conversely, gravitational attraction weakens progressively if the parameter falls below 1, causing the expansion rate to approach a constant asymptotically, thereby representing the open Universe model[19]. Consequently, to maintain a density parameter so proximate to 1 at this epoch, its value must have been exceedingly close to unity during the early stages of the Universe. For instance, during nucleosynthesis, approximately 1 second after the Big Bang, such precision is necessitated[9].

$$|\Omega_{\text{tot}}(t_{\text{nuc}}) - 1| \lesssim 10^{-16}. \tag{1.40}$$

The fine-tuning problem becomes particularly perplexing when we contemplate the remarkable proximity of the density parameter to unity, given the vast range of possible values it could have assumed. This challenge is intricately linked with the flatness problem in cosmology. The flatness problem revolves around the remarkable observation that the Universe appears to be flat on large scales, which is unexpected given the natural evolution of cosmic densities over time. According to the Friedmann equations, even the slightest deviation from a critical density would have led to a vastly different geometry of the Universe.

Hence, the fact that our Universe exhibits such close-to-flat behavior suggests a need for an explanation. In the forthcoming chapter, we will explore how inflation offers a compelling solution, shedding light on the mechanisms that might have rendered our Universe flat with such precision.

## 1.3 Problem Statement

Given the enigmatic nature of these phenomena, it became imperative to devise a mechanism capable of explaining why the Universe begins with seemingly improbable conditions. Before introducing inflation theory, these conditions were regarded more as mere curiosities. While working within the framework of the Big Bang theory, they were often accepted as basic assumptions. However, as cosmology advanced, it became increasingly evident that such assumptions were insufficient to satisfactorily explain the uniformity and homogeneity observed in the Universe on large scales. Inflation emerges as a solution to this challenge by providing a detailed mechanism that explains the Universe's initial conditions and establishes causal relationships between various aspects of its evolution.

Among the conceptual problems addressed by inflation are flatness and the cosmological horizon, thus resolving two of cosmology's most prominent puzzles. Furthermore, this theory significantly contributes to our understanding of the formation of large-scale structures in the Universe, as well as the observed heterogeneity in the temperature spectrum of the Cosmic Microwave Background (CMB). In the next chapter, we will delve deeper into the mechanism and definition of inflation theory, exploring how it has transformed our understanding of the Universe in its early stages and subsequent evolution.

Within the field of inflation research, the analysis of different inflation potentials is one of the primary areas of study. Through these analyses, several models have been developed to explain the characteristics of the early Universe and the observable consequences of this era. The predominant method of studying and contrasting these potentials with observations is through perturbation theory, which perturbs the conventional metric of the Universe. From this analysis, scalar and tensor perturbation spectra are derived, which are observables obtained from cosmological experiments in projects such as COBE or Planck[20].

By comparing observational parameters and observables derived from the study of potentials, we can restrict certain models and dismiss others if the data is accurate enough. The following sections of this work will scrutinize the Higgs model in particular detail.

## 1.4 General and Specific Objectives

The objective is to conduct a comprehensive investigation of the Higgs model and its role in the theory of cosmological inflation. This investigation will involve analyzing the dynamics of the model, its observational implications, and its alignment with relevant empirical data, such as those provided by the Planck mission. The ultimate goal of this investigation is to gain a deeper understanding of the Higgs model and its potential implications for our understanding of the early universe and the mechanism of inflation. By achieving this objective, we can further advance our understanding of fundamental physics and cosmology.

1. Explore the motivations and foundations of cosmological inflation: Delve into a comprehensive understanding of the issues addressed by inflation theory and its relation to the shortcomings of the standard Big Bang theory, such as flatness and horizon problems.

2. Characterize the dynamics of the Higgs model during inflation: Utilize the slow-roll approximation and numerical methods to investigate how the Higgs model drives inflationary expansion, focusing on the evolution of scalar fields and their implications for the generation of cosmological structures.

3. Analyze the observational predictions of the Higgs model: Compute and assess the predictions of the Higgs model for key cosmological observables, such as the scalar spectral index (nS), the tensor-scalar ratio (r), and the scalar power spectrum (PS), comparing them with available observational data.

4. Evaluate the viability and relevance of the Higgs model: Critically analyze the obtained results to determine the viability and relevance of the Higgs model in the context of contemporary cosmology, considering its ability to address identified cosmological issues and its alignment with observational data.

For that the following work has been divided into four chapters. The first chapter of this work plays a crucial role in introducing fundamental concepts necessary for comprehending subsequent analyses. Not only that, but it also sheds light on the Hot Big Bang Theory, which has fascinated scientists for decades. Through a detailed discussion of some of its puzzles and fine-tuning problems, this work presents a persuasive argument for developing the inflationary theory. The second chapter provides a deeper explanation of inflation as well as perturbation theory and outlines all the numerical considerations taken into account for the computational approach employed in this study. The third chapter presents the results, which are separated into background calculations and perturbation calculations, along with a discussion of the approaches used for analysis: numerical and slow-roll. Finally, the last chapter summarizes the topics covered in this work, along with the insights gleaned from the analysis of the Higgs model.

The Einstein summation convention is utilized throughout this work. Greek indices range from 0 to 3, and Latin indices range from 1 to 3. Understanding the metric signature $(-,+,+,+)$ is essential to grasp the fundamentals of advanced physics.

# Chapter 2

# Methodology

## 2.1 Inflation

According to the Big Bang theory, we start with the problem of the observable universe being too large. However, what would happen if our Universe were a very small primordial sphere for a tiny fraction of a second? This supposition could solve the horizon, flatness problems, magnetic monopoles, structures, etc [16]. Subsequently, exponential growth leads to the starting point of the traditional Big Bang theory. Guth proposes that the nascent Universe is filled with a "very pure substance" occupying all space, a field he named inflaton. However, this field's density does not increase in the same way as matter would. In other words, as space expands and contains matter, the density decreases. Yet, the same does not occur with the inflaton field. As space grows, the inflaton field also expands, maintaining its density. This peculiar behavior causes the Universe to grow exponentially, doubling its size every $10^{-34}$ seconds until it reaches its true state of equilibrium [11]. From that point onwards, the Universe can expand much more slowly, according to the predictions of the Big Bang theory. Liddle [11] develops his calculations mathematically for the expansion of the Universe as follows:

$$\ddot{a}(t) > 0. \tag{2.1}$$

When we refer to inflation, we usually mean a drastic expansion, that is, an exponential expansion. However, we do not usually specify what the expansion is happening with respect to. Therefore, we propose an alternative expression to which we can give a physical interpretation.

$$\frac{d}{dt}\frac{H^{-1}}{a} < 0. \tag{2.2}$$

According to Liddle[11], $\left(\frac{H^{-1}}{a}\right)$ is the comoving Hubble length. This is one of the most important characteristic scales of the features related to the expansion of the Universe, which is expected to decrease over time.

Now we must assume that the work will be carried out within the framework of the general relativity theory; for that, it can be evaluated in the equation of acceleration(1.12), as follows:

$$\rho + 3p < 0, \tag{2.3}$$

We must take the energy density $\rho$ as positive, thereby compelling the pressure $(p)$ to take negative values to satisfy our condition, considering that this is not affected by the curvature of the Universe,

$$p < -\frac{\rho}{3}. \tag{2.4}$$

Assuming our Universe as a perfect fluid with negative pressure, we can find the solution to the Friedman equation[9] (1.17) as follows:

$$a(t) = a_0 e^{Ht} \tag{2.5}$$

Furthermore, it is necessary to calculate the scale factor to the base of the number representing the exponential growth of the Universe during certain key periods, in our context related to inflation, also called the amount of inflation. This concept is used to describe how much the Universe has grown from the beginning of inflation to the present moment. A large value of e-folding N indicates significant expansion during inflation, contributing to explaining the homogeneity and isotropy observed in the large-scale Universe.

$$N(t) \equiv ln\frac{a(t_{end})}{a(t_{ini})}, \tag{2.6}$$

Now we must define a scalar field in accordance with the energy density and pressure of our Universe, which could take the form $\phi(\bar{x}, t) = \phi(t)$ so that it aligns with the homogeneity of our Universe so that we can express our scalar fields in the following way:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{2.7}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{2.8}$$

so we can see that $\dot{\phi}$ belongs to the kinetic part, and $V(\phi)$ to the potential part. If we make the assumption (necessary condition) of negative pressure, the potential part should dominate over the kinetic part from the equation (2.8). Therefore, inflation will exist only if this condition is met. If we replace these equations in (1.10), (1.12), and (1.16). With this, we will obtain the equations of motion for a Universe according to the FLRW metric during the inflation process, and this, in turn, will be governed by a scalar field with the following equations:

$$H^2 = \frac{8\pi G}{3}\left[V(\phi) + \frac{1}{2}\dot{\phi}^2\right], \tag{2.9}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left[\dot{\phi}^2 - V(\phi)\right], \tag{2.10}$$

$$\ddot{\phi} + 3H\phi + \frac{\partial V}{\partial \phi} = 0 \tag{2.11}$$

Now, we will discuss how inflation solves the horizon and flatness problems that arise from the classic Big Bang theory.

### 2.1.1 Addressing the horizon problem

The solution to the horizon problem is directly related to the amount of inflation, that is, the e-folding N, which we can extract from the equation (1.33). To do this, we rewrite the equation as $\Delta\Omega > 4\pi\delta$ in order to obtain a last scattering surface very isotropic[17]. After taking into account the considerations, we can express it as

$$N \gtrapprox -4 + \ln z_{end}. \tag{2.12}$$

This outcome indicates the presence of a specific quantity of e-foldings at which the criteria for resolving the horizon problem are met. Given that the redshift at the conclusion of inflation is on the order of approximately $10^{26}$, the corresponding number of e-foldings would be:

$$N \gtrapprox 55. \tag{2.13}$$

### 2.1.2 Addressing the flatness problem

To solve the flatness problem, we can rewrite the Friedman equation as an equation with density parameter $\omega$, in our case, omitting the cosmological constant.

$$\omega - 1 = \frac{K}{a^2 H^2} \tag{2.14}$$

.

If the density parameter of the Universe, $\omega$, varies over time, we can establish that the Universe exhibits a geometry different from Euclidean; however, currently, the $\omega$ parameter is very close to one, so we assign it a flat geometry. If we go back to the early Universe, for instance, during nucleosynthesis, we obtain values very close to one.

$$|\omega(t_{nuc}) - 1| \lesssim 10^{-16}. \tag{2.15}$$

We can see that the value is closer to one, so if we go back even further in time, the value of $\omega$ should be even closer to 1. This implies that the density parameter of the Universe, $\omega$, has undergone minimal variation, indicating that the Universe exhibits the same geometry as it does today, namely, Euclidean geometry.

## 2.2 Scalar field

According to equation (2.4), we deduce that the Universe's required material must possess certain special properties, specifically referring to negative pressure. Furthermore, the presence of a large cosmological density can pose a problem, as during the inflationary period, the energy density does not decrease with the creation of space. This discrepancy with observations contradicts the cosmological constant. Hence, we need the Universe to mimic a fluid with the mentioned characteristics, and this is achieved through a scalar field, expressed by the following Lagrangian[2].

$$\mathcal{L} = \frac{1}{2}\partial^{\mu}\partial_{\mu}\phi - V(\phi) = \frac{1}{2}\left[\phi^2 - \Delta^2\phi - V(\phi)\right] \tag{2.16}$$

Now we must relate the energy-momentum tensor through Noether's theorem, and in this way, we obtain:

$$T^{\mu\nu} = \partial^{\mu}\phi\partial^{\nu}\phi - g^{\mu\nu}\mathcal{L}, \tag{2.17}$$

Furthermore, as we are in a very early period of the Universe, we must also consider that the processes occurring at this epoch must involve very high energies. Therefore, we need to turn to fundamental physics to understand this process better. Hence, we will define the action using our Lagrangian and the scalar field. This field should be decoupled from other fields, so $V(\phi)$ will be a free function. Therefore, our equation to describe the action can be expressed as follows[17]:

$$S = -\int d^4x\,\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right]. \tag{2.18}$$

A significant portion of the literature typically employs a specific model known as a one-scalar field formulation, as described in[21]. In this context, "scalar" refers to a field with spin-0, similar to the Higgs boson.

Therefore, we can describe an action that relates the properties of our scalar field with the notation $\phi(t, x)$, incorporating both position and time. However, due to the symmetry of the energy-momentum tensor, it will lead to having only temporal dependence. Hence, we will refer to this field as the inflaton field. Using the relation $T^0_0 = \rho$, we obtain the time-time component belonging to the stress-energy tensor.

$$\rho = \frac{1}{2}(\dot{\phi})^2 + V(\phi). \tag{2.19}$$

Similarly, if we expand the space-space part using the relation $T^i_j = -p\delta_j i$, and also add the potential and kinetic terms, we can obtain

$$p = \frac{1}{2}(\dot{\phi})^2 - V(\phi). \tag{2.20}$$

With these new properties in hand, it's time to evaluate the Friedmann equation and the energy conservation equation, (1.17) and (1.16), respectively, obtaining the following results:

$$H^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right], \tag{2.21}$$

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0, \tag{2.22}$$

Here, $V'$ denotes the derivative of the potential with respect to the field $\phi$. The initial equation is commonly referred to as the Friedmann equation, while the second is known as the Klein-Gordon equation[22]. Together, this pair of expressions is termed the equations of motion for a Universe propelled by a scalar field.

In simpler terms, the condition for inflation to occur is that the rate of change of the scalar field, represented by $\dot{\phi}^2$, should be less than the potential energy, $V(\phi)$. This condition is easily met when the potential energy is sufficiently flat. For convenience, we will now drop the subscript '0'.

To understand the behavior of the potential, we can observe the scheme in the figure 2.1.

## 2.3 Slow-Roll

With the Friedmann(1.17) and Klein-Gordon(2.11) equations obtained in the previous section, we turn to the slow-roll approximation to derive the evolution of our field $\phi(t)$ and the scale factor a(t), as this

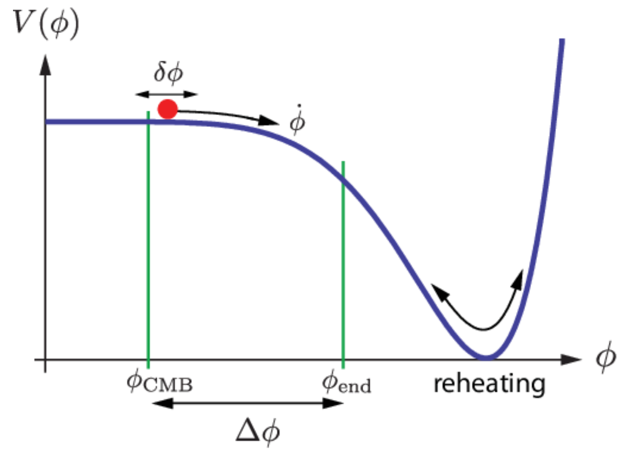



Figure 2.1: Schematic inflationary process[2].

approximation significantly simplifies the calculations. We can observe that the behavior of our equation (2.22) is similar to that of a harmonic oscillator with friction. In our equation, we can proportionally relate the Hubble parameter H to the friction term in a harmonic oscillator. As we know from the harmonic oscillator, if the value of the friction term is considerably high, the system will end up damping. Therefore, we can neglect the term $\ddot{\phi}$[23], leading us to the following approximations:

$$H^2 \simeq \frac{1}{3}V(\phi), \tag{2.23}$$

$$3H\dot{\phi} \simeq -V'. \tag{2.24}$$

Where the potential $V'$ is the derivative of $V$ with respect to the field $\phi$. In order to achieve the desired inflation, it is necessary to define certain parameters in the slow-roll approximation. To do this, we will classify them into two types of parameters. The first ones are called Hubble flow parameters, which are defined in terms of the Hubble parameter that evolves along with time[24]. Thus, we can express them as follows:

$$\epsilon_{\text{H}}(\phi) = \left[\frac{H'(\phi)}{H(\phi)}\right]^2, \tag{2.25}$$

$$\eta_{\text{H}}(\phi) = \frac{H''(\phi)}{H(\phi)}. \tag{2.26}$$

Likewise, the slow-roll parameters are formulated based on the potential of our field, reflecting how the evolution of the scalar field potential contributes to the dynamics of the inflationary process.[25 26]:

$$\epsilon_{\mathrm{V}}(\phi) \equiv \frac{1}{2}\left[\frac{V'(\phi)}{V(\phi)}\right]^2, \tag{2.27}$$

$$\eta_{\mathrm{V}}(\phi) \equiv \frac{V''(\phi)}{V(\phi)}. \tag{2.28}$$

These final equations demand not only knowledge of the inflation potential but also information about how it evolves over time. In this work, both approaches are employed, with the second one being utilized to extract field values at the conclusion of inflation. Meanwhile, the first approach is harnessed for the computation of observable quantities within the slow-roll paradigm. Specific conditions for inflation in terms of these parameters are established, thereby enhancing our understanding of inflationary processes.[27]:

$$\epsilon(\phi) << 1, \tag{2.29}$$

$$|\eta(\phi)| << 1. \tag{2.30}$$

For this reason, we establish that the violation of one of these conditions would imply the end of inflation[28], then:

$$\epsilon(\phi_{\mathrm{end}}) - 1 = 0, \tag{2.31}$$

$$|\eta(\phi_{\mathrm{end}})| - 1 = 0. \tag{2.32}$$

## 2.4 Perturbation Theory

The theory of perturbations in inflationary cosmology is a mathematical and conceptual framework that addresses small-scale quantum fluctuations in fundamental fields, such as the inflaton field, during the inflationary epoch of the Universe. During inflation, there are significant amplifications of these quantum fluctuations, which can subsequently evolve and give rise to anisotropies and inhomogeneities in matter density in the observable Universe.

Perturbation theory utilizes tools from quantum mechanics and quantum field theory to quantify and analyze how perturbations in fundamental fields evolve over cosmic time. These perturbations serve as seeds for the large-scale structures observed in the present-day Universe[29], such as galaxies and galaxy clusters.

To better understand these perturbations, cosmological perturbation theory has been developed to study the evolution and origin of small perturbations from homogeneous and isotropic solutions[21].

In inflationary cosmology, vector potentials are discarded because they do not significantly contribute to the inflationary mechanism[25]. Cosmic inflation is based on the exponential accelerated expansion of the Universe driven by the inflaton scalar field[30]. During this process, the focus is primarily on scalar and tensor perturbations associated with the inflaton.

Scalar perturbations are responsible for density fluctuations that later evolve to form observed structures in the Universe, such as galaxies and galaxy clusters. Tensor perturbations, on the other hand, are related to gravitational waves.

Vector perturbations, while they can exist, do not play a fundamental role in the inflationary mechanism. Therefore, they are discarded or considered negligible to explain cosmic observations within the inflationary framework. This simplifies the analysis and understanding of relevant phenomena during inflation, allowing a focus on key aspects of the inflationary model.

The analysis is conducted by considering coupled linear differential equations for each type of perturbation. This approach allows the study of one kind of perturbation while neglecting the effects of others. It also enables the examination of perturbations independently from background solutions. We need a quantity that can vary without affecting observable physical outcomes to achieve this. An important example is the initial value of the inflaton field, which drives the exponential expansion of the universe during inflation. Different choices of this initial value do not alter observable outcomes, simplifying calculations and theoretical analysis of cosmic inflation. To achieve this, gauge variables[31] and perturbations are defined, such that in the absence of perturbations, the system returns to its usual background state[21].

In this way, we consider a perturbed Friedmann-Robertson-Walker (FRW) metric for a flat Universe under the longitudinal gauge[29]. The metric's line element is given by:

$$ds^2 = -\left[1 + 2\Phi(t,x)\right] dt^2 + a^2\left[1 - 2\Psi(t,x)\right]\delta_{ij}dx^i dx^j, \tag{2.33}$$

Now, we need to solve the Einstein field equation for this perturbed metric.

$$\delta G^{\mu}_{\nu} \equiv \delta R^{\mu}_{\nu} - \frac{1}{2}\delta^{\mu}_{\nu}\delta\mathcal{R} = 8\pi G\delta T^{\mu}_{\nu}, \tag{2.34}$$

where $\delta$ represents the perturbation. Then we can consider the following matrix with the metric:

$$g_{\mu\nu} = \begin{pmatrix} -1 - 2\Phi(t,x) & 0 & 0 & 0 \\ 0 & a^2\left[1 - 2\Psi(t,x)\right] & 0 & 0 \\ 0 & 0 & a^2\left[1 - 2\Psi(t,x)\right] & 0 \\ 0 & 0 & 0 & a^2\left[1 - 2\Psi(t,x)\right] \end{pmatrix}. \tag{2.35}$$

As we can observe, the elements of the covariant metric are presented on the diagonal of the matrix (2.35), and if we were to obtain the elements of the contravariant metric, it would only be necessary to find the inverse value of each element of the diagonal matrix.

$$g^{00} = -\frac{1}{1+2\Phi}, \tag{2.36}$$

$$g^{ii} = \frac{1}{a^2(1-2\Psi)}, \tag{2.37}$$

If we now consider the values of $\Phi$ and $\Psi$ as small, we can make an approximation as follows:

$$g^{00} = -1 + 2\Phi, \tag{2.38}$$

$$g^{ii} = \frac{1}{a^2}(1+2\Psi). \tag{2.39}$$

Tapia and Rojas[29] conduct a detailed calculation to explicitly determine the perturbed values of the Einstein tensor $G^{\mu}_{\nu}$ and the energy-momentum tensor $T^{\mu}_{\nu}$, up to the first order. This results in the derivation of the following set of equations:

$$\nabla^2\Phi - 3\mathcal{H}\Phi' - \left(\mathcal{H}' + 2\mathcal{H}^2\right)\Phi = \frac{3}{2}l^2\left(\varphi_0'\delta\varphi' + V_\phi a^2\delta\varphi\right), \tag{2.40}$$

$$\Phi' + \mathcal{H}\Phi = \frac{3}{2}l^2\varphi_0'\delta\varphi, \tag{2.41}$$

$$\Phi'' + 3\mathcal{H}\Phi' + \left(\mathcal{H}' + 2\mathcal{H}^2\right)\Phi = \frac{3}{2}l^2\left(\varphi_0'\delta\varphi' - V_\varphi a^2\delta\varphi\right), \tag{2.42}$$

Where $\mathcal{H} = \frac{a'}{a}$ and $l^2 = \frac{8\pi G}{3}$. Additionally, we need to find the relationship between $H$ and $\mathcal{H}$ through its relation $H = \frac{\mathcal{H}}{a}$. The terms $\varphi_0$ and $\delta\varphi$ are part of the definition of the perturbed inflaton field.

$$\varphi(t,x) = \varphi_0(t) + \delta\varphi(t,x), \tag{2.43}$$

When using $\varphi_0$ as the background field and $\delta\varphi$ as the linear perturbation, along with $V_\varphi$ representing the derivative of our potential $V$ with respect to the inflaton field, the combination of the previously outlined perturbation equations yields the following expression:

$$\Phi'' - \nabla^2\Phi + 2\left(\frac{a}{\varphi_0'}\right)'\left(\frac{a}{\varphi_0'}\right)^{-1}\Phi' + 2\varphi_0'\left(\frac{\mathcal{H}}{\varphi_0'}\right)'\Phi = 0, \tag{2.44}$$

in the same way,

$$\sigma'' - \nabla^2\sigma - z\left(\frac{1}{z}\right)''\sigma = 0. \tag{2.45}$$

There will be specific power spectra for scalar and tensor perturbations, as outlined in the research conducted by Meza et al. in 2021[32].

$$P_{\rm S}(k) = \lim_{kt\to\infty} \frac{k^3}{2\pi^2}\left|\frac{u_{\rm k}(t)}{z_{\rm S}(t)}\right|^2, \tag{2.46}$$

$$P_{\rm T}(k) = \lim_{kt\to\infty} \frac{k^3}{2\pi^2}\left|\frac{v_{\rm k}(t)}{a(t)}\right|^2, \tag{2.47}$$

And along with these variables, there emerges another one that proves equally beneficial in the task of constraining inflationary models, namely, the tensor-scalar ratio[33] denoted as $r$:

$$r(k) = 8\frac{P_{\rm T}(k)}{P_{\rm S}(k)}. \tag{2.48}$$

Similarly, spectral indices are defined for both scalar and tensor perturbations[33]:

$$n_{\rm S}(k) = 1 + \frac{d\ln P_{\rm S}}{d\ln k}, \tag{2.49}$$

$$n_{\rm T}(k) = \left(\frac{d\ln P_{\rm T}}{d\ln k}\right). \tag{2.50}$$

However, an alternative method for deriving the expression of the value for $P_{\rm S}(k)$ is often employed when dealing with observational data or when the investigation is not influenced by the selection of a specific potential. This method treats the scalar spectrum as a generic function, aiming to model and fit its shape as a power law[34]:

$$\log P_{\rm S}(k) = \log A_{\rm S} + (n_{\rm S}-1)\log\left(\frac{k}{k_*}\right) + \frac{1}{2}\alpha_{\rm S}\log^2\left(\frac{k}{k_*}\right) + ..., \tag{2.51}$$

Here, $A_{\rm S}$ represents the spectrum's amplitude, $\alpha_{\rm S}$ denotes the running parameter, and $k_*$ is the pivot scale. Similarly, there is a corresponding expression for the tensor spectrum as well[20]:

$$\log P_{\rm T}(k) = \log rA_{\rm S} + n_{\rm T}\log\left(\frac{k}{k_*}\right) + ..., \tag{2.52}$$

## 2.5 Higgs Potential

The expression of the Higgs potential model plays a significant role in addressing fundamental phenomena related to the Higgs field in the early Universe. This mathematical formulation provides theoretical tools to explore how the Higgs field influenced phase transitions and particle mass generation during crucial moments, such as cosmic inflation. Considering the Higgs potential aims to understand how the field's properties contribute to the formation and evolution of the Universe in its early stages, Providing valuable perspectives on the foundational mechanisms that shaped the structure and dynamics of the cosmos on a large scale, the mathematical expression is[35]:

$$V(\phi) = M^4 \left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 , \tag{2.53}$$

The Higgs field potential can be chosen due to the particular properties that make it suitable for describing the dynamics of the inflaton field during the inflationary period. The Higgs field potential has several desirable features that make it useful in this context:

Electroweak symmetry: The Higgs field potential is linked to the breaking of electroweak symmetry in particle physics, providing a natural connection between cosmic inflation and the fundamental forces of nature.

High-energy stability: The Higgs field potential can be stable enough at high energies to allow for prolonged and successful inflation, ensuring that the universe undergoes exponential expansion for a sufficiently long time to address cosmological problems.

Production of quantum fluctuations: During inflation, quantum fluctuations in the inflaton field become the seeds of the structures observed in the universe, such as galaxies and large-scale structures. The Higgs field potential can efficiently generate these fluctuations and produce a spectrum of perturbations consistent with observations.

Consistency with observational data: By choosing a suitable Higgs field potential and adjusting its parameters, one can obtain an inflationary model consistent with cosmological observations, such as the cosmic microwave background radiation and the distribution of galaxies.

The graph of the Higgs potential in the inflationary theory with the slow-roll method visually represents the smooth and gradual evolution of the Higgs field during the cosmic inflation period. In this depiction, the gentle slope of the potential reflects the small rate of change of the Higgs field as it evolves slowly. The slow-roll method allows for calculating key parameters, such as the spectral index and the tensor-scalar ratio, characterizing cosmic perturbations generated during inflation. These parameters have observational implications and offer valuable insights into the connection between the Higgs potential and observable features in the current Universe, enriching our understanding of the fundamental processes that shaped the cosmos.

The choice of the potential's shape, as seen in Fig. 2.2, is made to ensure an exceptionally flat potential at the onset of inflation. This configuration allows for what is commonly known as a slow-roll phase, characterized by the dominance of the potential term over the kinetic term within this range. Additionally,

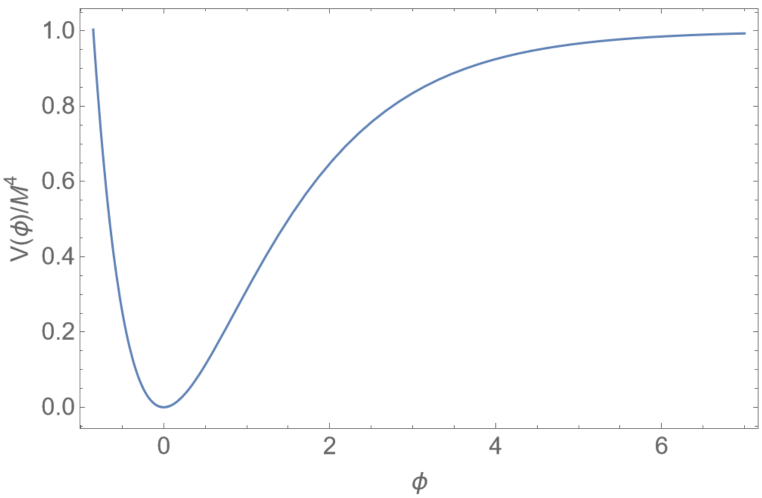



Figure 2.2: Higgs potential.

it is worth noting that $\phi$ in Fig. 2.2 is dimensionaless, indicating it is expressed without any specific physical unit.

# Chapter 3

# Results & Discussion

To provide a clear and organized understanding of the results obtained in this study, we have categorized our findings as follows:

First, we present the calculations performed for the slow-roll parameters. These calculations yield crucial values such as the field quantities at the end and beginning of inflation, which are essential parameters for determining the background dynamics. The results of these calculations are detailed in the subsequent section.

In addition, we provide solutions for both the slow-roll approximation and the numerical results. Additionally, we present the scalar and tensor power spectra, along with the relationship between observable values, including the spectral index and tensor-scalar ratio. It is important to note that all these results are based on both the slow-roll approximation and numerical computations.

Furthermore, it is worth mentioning that all calculations were conducted using a conventional laptop equipped with an 11th Gen Intel(R) Core(TM) i7-1165G7 processor operating at 2.80 GHz. The computations were performed using Mathematica 13.03 software[36].

## 3.1 Slow-Roll Parameters

To calculate the slow-roll parameter $\epsilon_1$ analytically, we can obtain the following results:

$$\epsilon = \frac{1}{2}\left[\frac{V'(\phi)}{V(\phi)}\right]^2, \tag{3.1}$$

Substituting and deriving respectively with the values of our Higgs potential, we obtain the following result:

$$\epsilon = \frac{4}{3}\left(-1 + e^{\sqrt{\frac{2}{3}}\phi}\right)^{-2}. \tag{3.2}$$

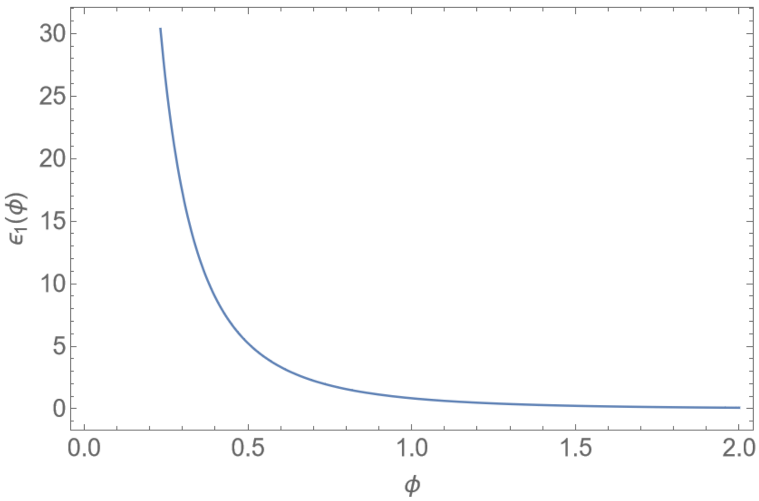



Figure 3.1: First slow-roll parameter $\epsilon$.

In order to obtain the values of $\phi_{\text{end}}$, it is necessary to assume that the Universe will be flat in the end, that we see in section (2), so we must set the value of epsilon to 1 and solve for the value of $\phi_{\text{end}}$.

$$\frac{4}{3}\left(-1 + e^{\sqrt{\frac{2}{3}}\phi}\right)^{-2} = 1 \tag{3.3}$$

We set the value of epsilon parameter $\epsilon$ to one to assume that at that moment, the Universe exhibits complete flatness at the end of the expansion.

Similarly, with our Higgs potential, we can now find the parameter $\eta$ according to the equation of $\eta_{\text{V}}(\phi)$, expressed as follows:

$$\eta_{\text{V}}(\phi) = \frac{V''(\phi)}{V(\phi)}, \tag{3.4}$$

Substituting and deriving respectively with the values of our Higgs potential, we obtain the following result:

$$\eta_{\text{V}}(\phi) = -\frac{4}{3}\frac{e^{\sqrt{2/3}\phi} - 2}{\left(e^{\sqrt{2/3}\phi} - 1\right)^2}. \tag{3.5}$$

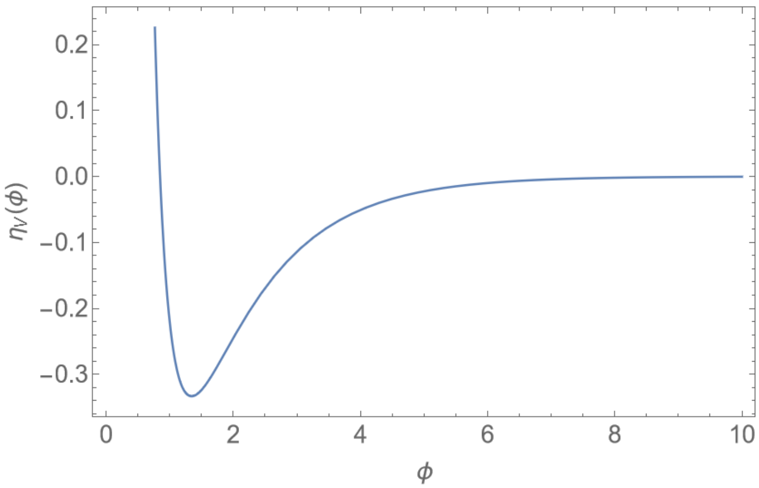



Figure 3.2: $\eta$ parameter respect to the field $\phi$.

In the figure (3.2) we can see the behavior of the parameter $\eta_{\mathrm{V}}(\phi)$ respect to the potential $\phi$

Now it is necessary to calculate the amount of inflation, as discussed in Section (2), with our specific field, that is, the Higgs field $\phi$. For this, we turn to the definition found in Liddle[11].

$$N = \int_{\phi}^{\phi_{\mathrm{ini}}} \frac{V(\phi)}{V_{\phi}(\phi)} d\phi. \tag{3.6}$$

With the values of our Higgs potential, and knowing that $\phi_{\mathrm{ini}}$ represents the values of $\phi$ at the beginning of the inflationary stage, and that $V_{\phi}$ is the derivative of the potential with respect to the field $\phi$, we obtain the following:

$$N = \int_{\phi}^{\phi_{\mathrm{ini}}} \frac{M^4(1 - e^{-\sqrt{2/3}\phi})^2}{2\sqrt{2/3}M^4 e - \sqrt{2/3}\phi(1 - e^{-\sqrt{2/3}\phi})} d\phi, \tag{3.7}$$

Carrying out the appropriate calculations, we obtain that the e-folding N, is:

$$N = \frac{3}{4} e^{\sqrt{2/3}\phi_{ini}} - \frac{1}{2}\sqrt{\frac{3}{2}}\phi_{ini} - \frac{3}{4} e^{\sqrt{2/3}\phi} + \frac{1}{2}\sqrt{\frac{3}{2}}\phi. \tag{3.8}$$

## 3.2 Slow-roll Solutions

Taking into account the previous results from Section 3.1, we will solve the equations governing the Universe using the slow-roll approximation.

$$H^2 \simeq \frac{1}{3}V(\phi), \tag{3.9}$$

$$3H\dot{\phi} \simeq -V_\phi. \tag{3.10}$$

We should not forget the following considerations $H = H(t)$ and $\phi = \phi(t)$, obtaining.

$$H^2(t) \approx \frac{1}{3}V\left[\phi(t)\right], \tag{3.11}$$

$$3H(t)\dot{\phi}(t) \approx -\frac{\partial V\left[\phi(t)\right]}{\partial \phi(t)}, \tag{3.12}$$

$$\left[\frac{\dot{a}(t)}{a(t)}\right]^2 \approx \frac{1}{3}V\left[\phi(t)\right], \tag{3.13}$$

$$3\left[\frac{\dot{a}(t)}{a(t)}\right]\dot{\phi}(t) \approx -\frac{\partial V\left[\phi(t)\right]}{\partial \phi(t)}. \tag{3.14}$$

Another important parameter that we must calculate is the value of M, for which we will use the $\delta_R$ with a value of $2.1 \times 10^{-9}$ according to [20], obtaining the following results:

$$M^4 = \frac{32\delta_R \pi^2 e^{2\sqrt{\frac{2}{3}}\phi}}{\left(-1 + e^{\sqrt{\frac{2}{3}}\phi}\right)^4}. \tag{3.15}$$

Where M is an independent parameter that serves to standardize the scalar power spectrum.

According to Section 2.1, we will use a value of $N = 60$ since it satisfies the expansion conditions required by inflationary theory. Consequently, we can determine the value of $\phi_{ini}$ at the beginning and $\phi_{end}$ at the end of the inflationary process, which we can express the following way:

$$\phi_{ini} = -\sqrt{\frac{3}{2}}\left(1 + \frac{2}{\sqrt{3}}\right) - 2\sqrt{\frac{2}{3}}N + \sqrt{\frac{3}{2}}Log\left(1 + \frac{2}{\sqrt{3}}\right) - \sqrt{\frac{3}{2}}\text{ProductLog}\left[-1, -e^{-\left(1+\frac{2}{\sqrt{3}}\right) - \frac{4N}{3} + Log\left(1+\frac{2}{\sqrt{3}}\right)}\right]. \tag{3.16}$$

Where the ProductLog function is the Lambert W function, this function is a feature available in the Mathematica[36] program. It allows for the computation of solutions to the equation ($z = W(z) \cdot e^{W(z)}$).

$$\phi_{end} = \sqrt{\frac{3}{2}}log\left(1 + \frac{2}{\sqrt{3}}\right). \tag{3.17}$$

With the approximate value of $\phi_{\text{end}} \approx 0.940178$ and $\phi_{\text{ini}} \approx 5.45315$, we now have both the exact and approximate values of $\phi$ at the beginning and end of the inflationary period. We have the tools to solve the equations of motion for our proposed Universe. These values will be used in the approximate slow-roll equation and subsequently employed in a numerical approximation.

The results of the equation using the slow-roll approximation are depicted in Figures 3.3 and 3.4, where we represent the temporal evolution of both $a(t)$ and $\phi(t)$, respectively. It's worth noting that time and its corresponding function $a(t)$ are expressed in Planck units.

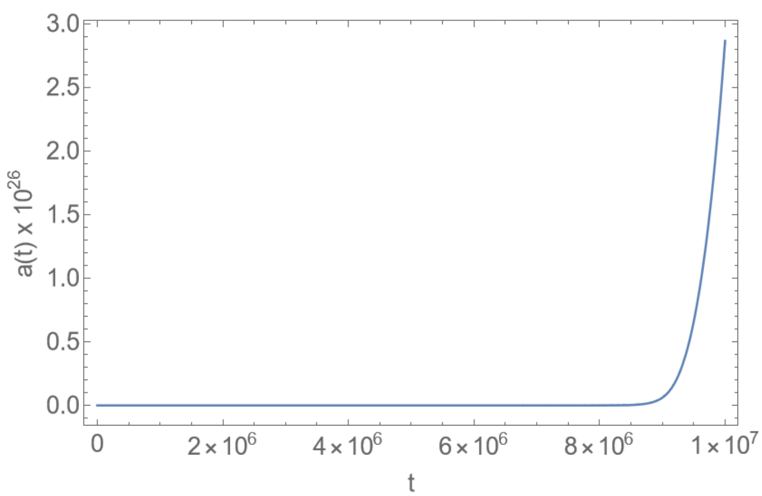



Figure 3.3: $a$ from slow-roll respect to time.

## 3.3 Power Spectra

The power spectrum is a crucial tool that characterizes the distribution of primordial quantum fluctuations in the early Universe. It provides essential information about the amplitude (3.1) and scale of perturbations that give rise to large-scale structures such as galaxies and galaxy clusters. The shape and features of the power spectrum offer valuable insights into the Universe's initial conditions during the inflationary period and serve as a key constraint for our model.

In the previous chapter, we can see the expressions for the power spectrum to obtain the observables, $ln(10^{10}A_S(k))$ and $n_S(k)$, whose results are shown in Table (3.1) compared with the results from Planck 2018 [20]. Additionally, we present the results in a figure (3.5) where we compare the power spectrum against k. We also showcase the same result in a semi-logarithmic graph.

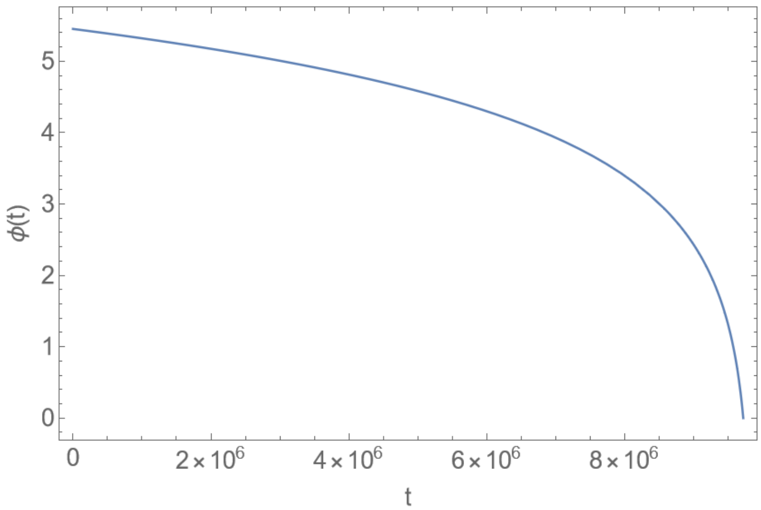


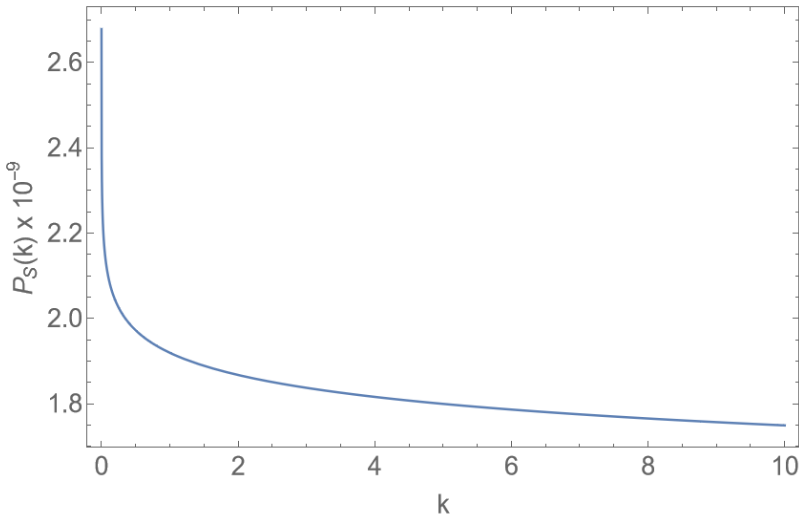

Figure 3.4: $\phi$ from slow-roll respect to time.

| Observable | Plank Results[20] | Higgs inflation |
|---|---|---|
| $ln(10^{10}A_S)$ | 3.040 | 3.0677 |
| $n_S$ | 0.9626 | 0.961705 |

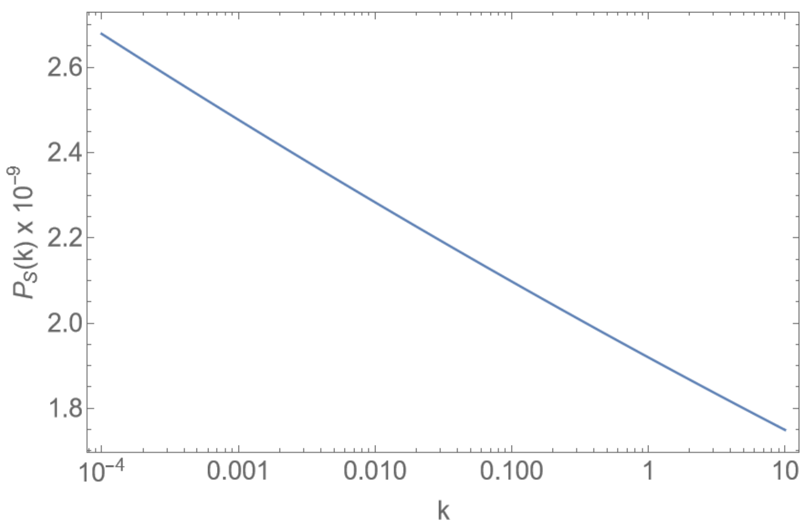
Table 3.1: Comparative values of Plank results vs our results.

Following the same path, we obtain the values of $P_T(k)$, which we present in figure 3.6; these figures are in linear and semi-log scale; furthermore, we can find the parameters $n_S$ and $n_T$, and just like the previous parameters, we graph them in the figure (3.7).

In our context, $n_S$ and $n_T$ are important parameters related to the scalar and tensor power spectra. $n_S$ refers to the spectral index of the scalar power spectrum. This parameter describes how the density fluctuation spectrum varies with scale (length or momentum). A value of $n_S$ close to 1 indicates nearly scale-invariant fluctuations, meaning that density fluctuations have approximately the same amplitude on all scales. However, if $n_S$ significantly differs from 1, it indicates scale dependence in the power spectrum, implying that fluctuations vary in amplitude at different scales. $n_T$ is the spectral index of the tensor power spectrum. This parameter describes the scale dependence of the primordial gravitational wave amplitudes. If $n_T$ is zero, it indicates no scale dependence in the tensor spectrum, meaning that gravitational waves have the same amplitude on all scales. However, if $n_T$ is non-zero, it indicates scale dependence in the




(a) $P_S(k)$ vs $k$.



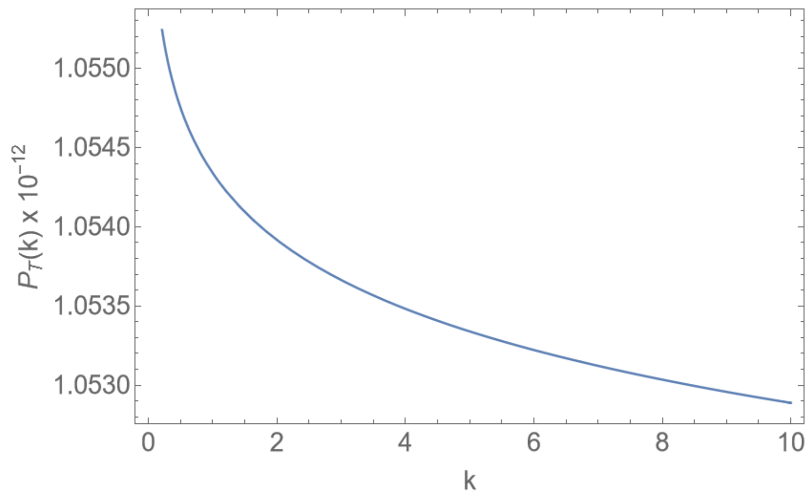

(b) Log Plot $P_S(k)$ vs $k$ .

Figure 3.5: Plots of Scalar perturbation spectrum for slow-roll approximation for $N = 60$.

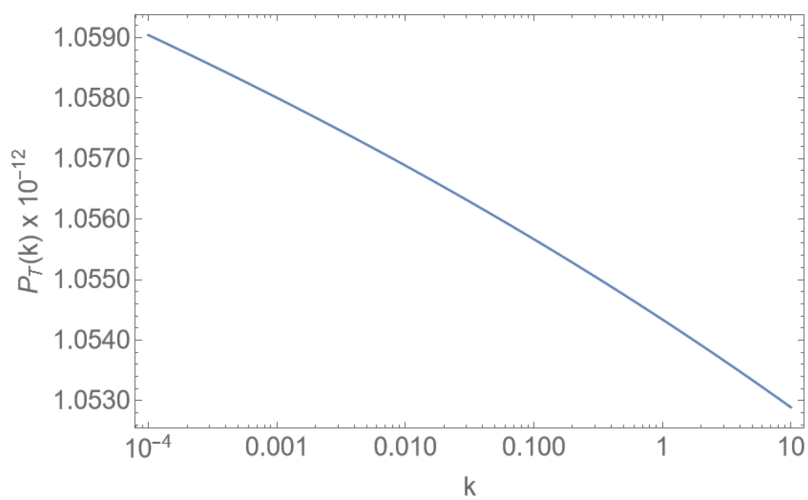
tensor spectrum, implying that gravitational waves vary in amplitude at different scales.

Finally, we can establish a relationship between the two spectra, scalar and tensorial, as follows:

$$r(k) = \frac{P_T}{P_S} = 16\epsilon, \tag{3.18}$$



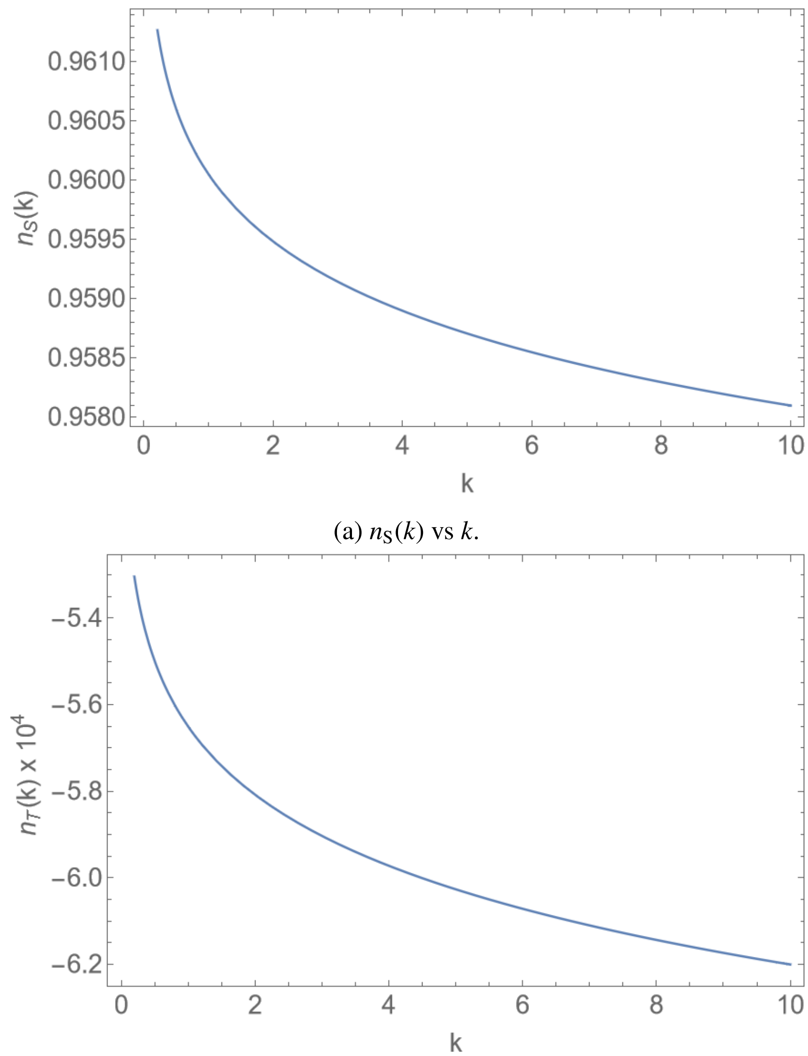

(a) $P_{\mathrm{T}}(k)$ vs $k$.




(b) Log Plot $P_{\mathrm{T}}(k)$ vs $k$ .

Figure 3.6: Plots of Tensor perturbation spectrum for slow-roll approximation with $N = 60$.

This equation implies that the ratio of the tensor power spectrum to the scalar power spectrum is equal to 16 times the value of the tensor-to-scalar ratio. Therefore, based on our scalar and tensor power spectrum values, the ratio value will be $r(k) = 0.00358893$.




(a) $n_S(k)$ vs $k$.

(b) $n_T(k)$ vs $k$ .

Figure 3.7: Plots of $n_S$ and $n_T$ with $N = 60$.

## 3.4 Numerical Solutions

Now, we want to obtain numerical results without approximation so that we will take the complete equations of motion for the Universe, that is:

$$H^2 = \frac{1}{3}\left[V(\phi) + \frac{1}{2}\dot{\phi}^2\right], \tag{3.19}$$

$$\ddot{\phi} + 3H\dot{\phi} = -V_\phi(\phi). \tag{3.20}$$

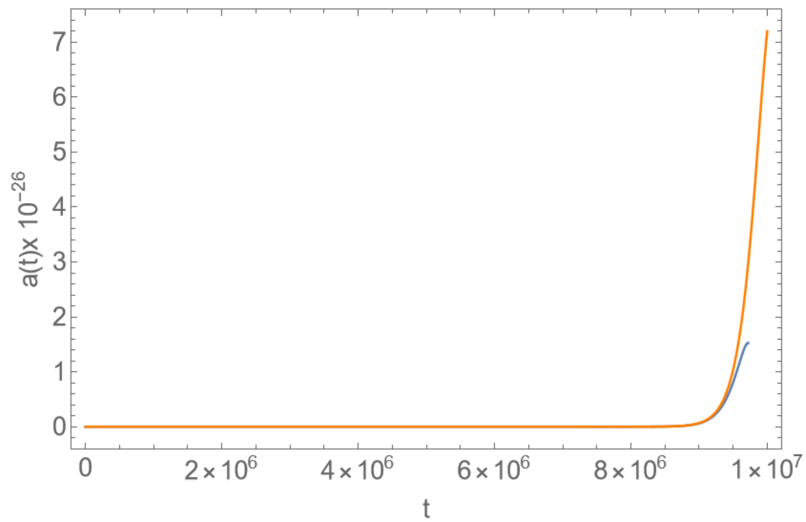
The Mathematica function $NDSolve$ was employed to compute the solutions for each function numerically: the inflaton field $\phi(t)$ and the scale factor $a(t)$. The initial conditions for these calculations were derived from the results of the slow-roll approximation presented in the previous section, evaluated at $t = 0$.

Figures 3.8a and 3.8b illustrate the temporal evolution of the scale factor $a$ and the inflation field $\phi$, respectively. Considering a number of e-foldings $N = 60$. Additionally, Fig. (3.8) provides a comparison between the numerical results (orange line) and those obtained from the slow-roll approximation (blue line). The figures demonstrate that the approximation holds well most of the time, exhibiting minimal discrepancies. However, as the cosmic time approaches the end of inflation, a noticeable separation between the slow-roll and numerical results becomes apparent.

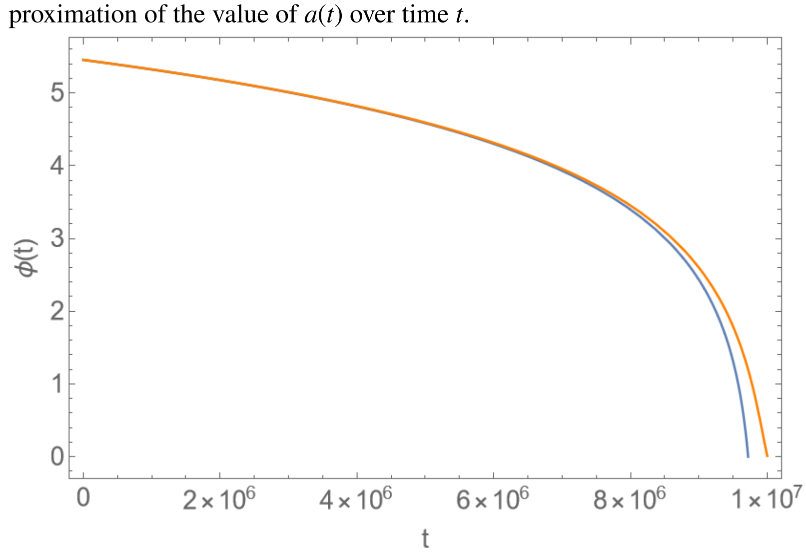
Finally, we can compare the scalar power spectrum using the slow-roll approximation (blue line) compared to the numerical calculation (orange line) in figure 3.9. Similarly, we observe the tensor power spectrum in figure 3.10; both figures are on a semi-logarithmic scale. According to figures 3.11 and 3.12, the values fall within the permissible ranges, thus confirming the viability of our slow roll model for the purpose of our research.

Finally, we take slow-roll values (10 values) to which we apply the absolute error and graph them, obtaining the following results for the scalar power spectrum $P_S(k)$ and the tensor power spectrum $P_T(k)$. The errors that arise are due to the approximations made with the slow-roll model; however, it is a fairly reliable model.

The discrepancy between the slow-roll model and the exact numerical approach based on the Higgs model in inflationary theory can be attributed to various fundamental reasons. On the one hand, the slow-roll model simplifies inflationary dynamics by assuming that the inflaton fields change slowly, potentially overlooking more dynamic and complex effects present during inflation. On the other hand, the exact numerical approach of the Higgs model considers the full dynamical equations of the Higgs field and its interaction with the inflaton field, allowing for a more precise description of early Universe evolution. The exhaustive exploration of a wide range of initial conditions and inflationary parameters in the numerical model may reveal effects not captured by slow-roll approximations. Ultimately, the discrepancy between both approaches underscores the importance of considering more complex effects and validating theoretical models with experimental data to understand the physical processes that shaped the early Universe fully.




(a) Comparative image of the numerical (orange) and slow-roll (blue) approximation of the value of $a(t)$ over time $t$.




(b) Comparative image of the numerical method (orange) and slow-roll (blue) of the value of $\phi(t)$ over time $t$.

Figure 3.8: Showing the numerical solutions (orange) and slow-roll (blue) of the background equations of motion for the scale factor $a$ and inflaton field $\phi$.

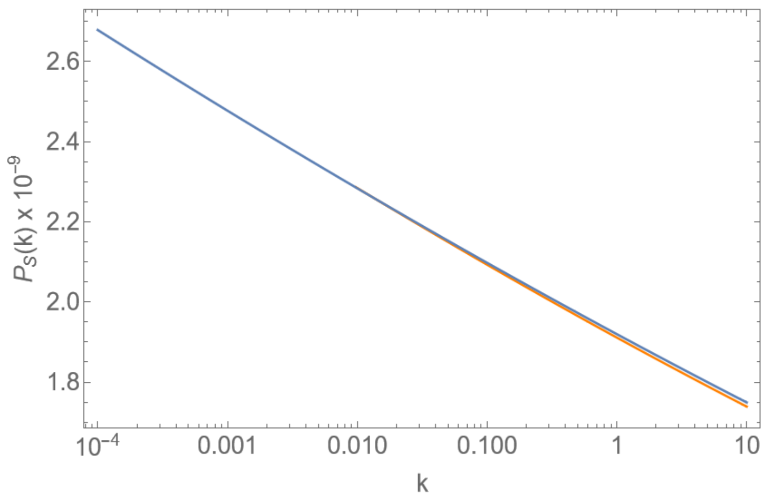



Figure 3.9: Numerical solution (Orange) and slow-roll solution (Blue) of $P_S$ in semi-log scale.

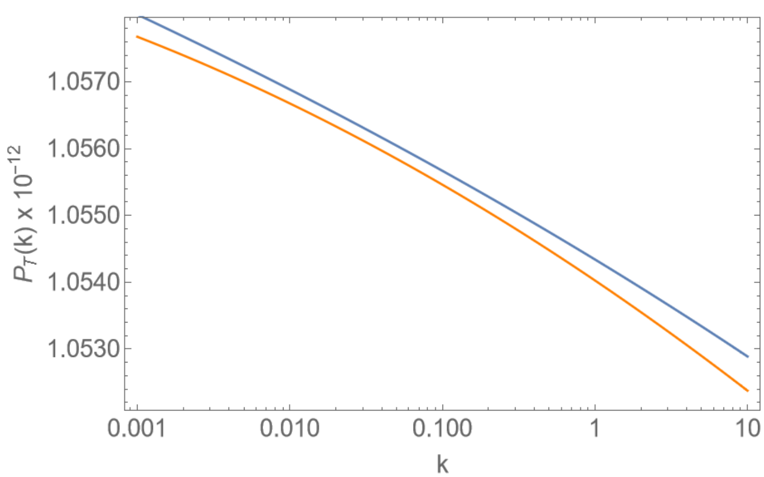



Figure 3.10: Numerical solution (Orange) and slow-roll solution (Blue) of $P_T$ in semi-log scale.

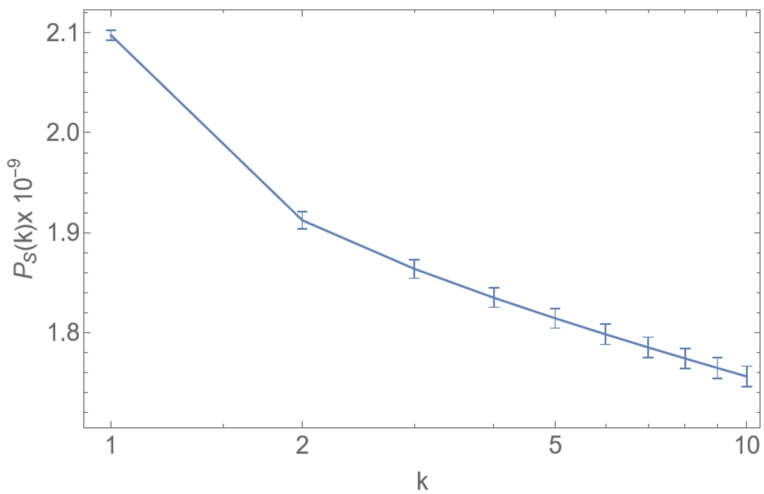



Figure 3.11: Absolute error applied to $P_S(k)$

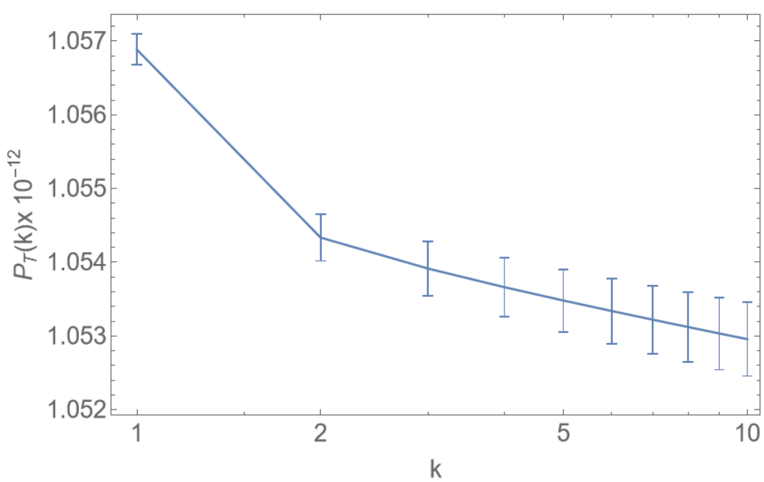



Figure 3.12: Absolute error applied to $P_T(k)$

# Chapter 4

# Conclusions

This study examines fundamental concepts necessary to delve into inflationary cosmology. The Hot Big Bang Model theory is introduced, and some of the issues this approach raises are discussed. Additionally, the equations primarily used in cosmology are developed and explained. Inflation is put forward as a resolution to these issues, as well as other questions that were still unresolved during the development of the theory. These include the correlation between the early Universe and large-scale formation and the inconsistencies in the distribution of temperatures in the CMB. If you're looking to understand inflationary models, the slow-roll approximation is an essential concept to grasp. By focusing on potential energy instead of kinetic energy in the equations of motion for the Universe, this approximation allows for a deeper understanding of the complex concepts at play. The analysis requires certain parameters to be defined, and some of the most crucial ones have been identified. Furthermore, we found the connection between theoretical and computational work with observations. The Higgs potential used in this study is presented in the same context.

The results chapter presents all information about the analysis conducted, both in the slow-roll approximation and numerical results. This section shows the background results. Characteristics such as the value of the field at the beginning and end of inflation are calculated using slow-roll parameters, graphs depicting the evolution of the inflaton field $\phi(t)$ and scale factor $a(t)$ are used to present solutions to the equations of motion of the Universe.

Additionally, functions describe the evolution over time. Using the provided definitions of the scalar and tensor power spectra, information is obtained for different scale values $k$ necessary to adjust these spectra according to the power law.

Finally, values of observables such as the spectral index $n_S(k)$ and the tensor-scalar ratio $r(k)$ are evaluated at a specific $k$. These calculations are carried out in both the slow-roll approximation and the numerical approach and are compared with the results of the Planck experiment. In conclusion, it is suggested that the Higgs model is consistent with observational results, and a range of parameters supporting this claim with a high confidence level are identified.

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