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TÍTULO: A MATHEMATICAL APPROACH TO UPGRADE SLUMS IN ECUADORIAN CITIES

Trabajo de integración curricular presentado como requisito para la obtención del título de Matemático

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Dedication

"To all the people whom for necessity have to live in slums."

Saulo Javier Ronquillo Guachamin

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"I thank my family for their continued support,
my teachers for sharing their knowledge with me,
my friends for giving me their backing
and all the citizens of Ecuador who allowed this dream called Yachay to begin."

Saulo Javier Ronquillo Guachamin

Resumen

Al mejorar la situación actual en los barrios marginales, el Ecuador estaría un paso más cerca de alcanzar los Objetivos de Desarrollo Sostenible para 2030. Hoy en día, existe la voluntad política para mitigar este problema pero, desafortunadamente, un mecanismo que busque una solución óptima con la mínima inversión no ha sido estudiado en Ecuador.

Brelsford et al. [1] propusieron un enfoque matemático para mejorar cualquier barrio marginal. Allí los autores aseguran que es la topología y no la geometría, la que dicta la forma esencial de las ciudades. Por ello, el crecimiento y/o planificación de los suburbios/-vecindarios se deberían centrar en cambiar la topología de las ciudades, independientemente de su geometría específica. Estos cambios en la topología se logran mediante la construcción de nuevas carreteras. Por lo tanto, uno puede hacer que cualquier barrio marginal tenga propiedades topológicas similares a un vecindario planificado con la creación de calles. Adicionalmente, por los escasos recursos económicos de muchos países es necesario encontrar la forma de elegir la mejor combinación posible de calles con el objetivo de obtener un barrio que sea topológicamente equivalente a un barrio planificado con el uso de la menor cantidad de recursos posibles.

En este trabajo se llevó a cabo un estudio del enfoque propuesto por Brelsford et al. Se realizó una replica de sus métodos en un ejemplo académico y se utilizó este enfoque en una aplicación de la vida real.

Keywords— Mejora de barrios marginales, teoría de grafos topológicos, optimización topológica, topología de la ciudad

Abstract

Helping to improve the current situation in Ecuadorian slums would be a step closer to achieving the Objectives of Sustainable Development by 2030. Nowadays, there is already the political determination to mitigate this problem but unfortunately, a mechanism that seeks an optimal solution with the minimum investment has not been studied in Ecuador.

A mathematical approach for upgrading any slum was proposed by Brelsford et al.[1]. There, the authors ensure that it is the topology and not the geometry, that dictates the essential shape of the cities. So that the growth and/or planning of the suburbs/neighborhoods should be focused on changing the topology of cities, regardless of their specific geometry. These changes in the topology are achieved through the construction of new roads. Therefore, one can make any slum has similar topological properties to a planned neighborhood with the creation of streets.

In this thesis, a study of the approach proposed by Brelsford et al. is done by replicating its methods in an academic example and using this approach in a real-life application.

Keywords — Slums upgrading, topological graph theory, topological optimization, topology of city

Contents

1. Introduction **2**

2. Basic Concepts **3**

2.1. Slums . . . 3

2.1.1. Definition and Properties . . . 3

2.1.2. Slums Upgrading . . . 4

2.1.3. Slums in Guayaquil . . . 5

2.2. Topological Graph Theory . . . 6

2.3. Topological Optimization . . . 9

3. Results **10**

3.1. Topology of a City . . . 11

3.1.1. Topology of the Access System . . . 11

3.1.2. Topology of City Blocks . . . 12

3.2. Minimal Neighborhood Reblocking . . . 16

4. Implementation **16**

4.1. An Academic Example . . . 16

4.2. *Ciudad de Dios* neighborhood in Guayaquil . . . 19

5. Conclusions **21**

References **22**

Appendices **24**

A. Code in Python **24**

B. Euler's Figures **28**

1. Introduction

The deurbanized growth of cities is a problem that has plagued Ecuador for a long time. Particularly, in the city of Guayaquil, you can find many marginal neighborhoods that lack adequate basic services[2]. This problem is so common in Ecuador that the government created the *Superintendencia de Ordenamiento Territorial, Uso y Gestión del Suelo* in 2016. The objective of this organization is to help citizens to have better access to a safe habitat[3].

Despite the efforts made by the government, the number of new constructions in informal settlements increases every year. This could be because although people living in informal settlements are in worse conditions than the ones living in formal urban areas, these people have better access to jobs located in the city than if they would live in places far from the city[4]. However, the inadequate infrastructure of informal settlements typically implies that their inhabitants do not have access to a better quality of life[5].

Since adequate urbanization increases the potential of human development and economic growth, among other things, many cities are willing to improve the planning of neighborhoods. Throughout history, there have been several approaches to improve the situation of slums in the world. Around the 1970s the preferred option was the relocation of residents to new public housing developments. However, given the large number of people living in slums, this solution required an exorbitant amount of resources, which made it an unfeasible option for developing countries[6].

Nowadays, one of the most recommended strategies by the United Nations Human Settlements Program (UN-Habitat) is to build new streets or change the traffic direction of them to improve the condition of the slums[6]. This is because the street network can improve the integration of the slums in two levels. Within the neighborhoods, the streets allow better integration of its inhabitants, increasing social interaction and increasing the economic development opportunities of the inhabitants in the place. And within the city, the streets allow the integration of the neighborhood with the rest of the metropolis through a physical integration to the urban transport network which benefits the city[6].

However, the development of new streets is often an expensive process, especially for developing countries. Consequently, it is necessary to plan the construction of new roads minimizing the investment. For this purpose, a mathematical approach seems to be a suitable tool to solve this problem[1].

Several researchers have analyzed this problem with different approaches. Some of them have tried to explain the shape of cities based on fractals, while others have used the ideas behind cellular automata to understand the complex system of how cities evolve[7]. The approach that is discussed here uses tools of topological graph theory to study the difference between planned neighborhoods and slums[1].

Graph theory has been used to better understand cities[8]. For instance, in 1735, Leonhard Euler solved what is known as the problem of the bridges of Königsberg[9]. This problem consists in how a citizen of Königsberg could travel the city crossing its seven bridges and return home. Euler quickly noticed that "this branch is concerned only with the determination of position and its properties; It does not involve distances, nor calculations made with them"[9]. Some historians believe that the paper of Königsberg is also the precursor of what is known as

Topology since the solution of this problem opened the way to the study of the Hamiltonian circuits, a topic that continues to be investigated these days[9].

In an article by Luis Bettencourt and Geoffrey West for the journal Nature, they indicate that "New York and Tokyo are, to a surprising and predictable degree, non-linear versions of San Francisco in California or Nagoya in Japan"[10]. This means that there are common properties between cities and that a city can be considered as an approximately scaled version of another. This same fact is used by Brelsford et al. because they identify that regardless of the appearance of the neighborhoods there are topological properties that vary if a planned neighborhood or a slum is analyzed[1]. Moreover, they show that it is possible to change the configuration of a slum to eliminate these differences with planned neighborhoods[1].

This work shows the development and use of an algorithm that tells us how road construction should be planned to upgrade slums. In this thesis, we implement and run this algorithm in a sector of *Ciudad de Dios*, a marginal neighborhood of Guayaquil, with the aim that this area is topologically equivalent to a planned neighborhood. Thus, the main objective is to analyze the mathematical approach proposed by Brelsford et al. [1] and replicate the method in an academic example and the *Ciudad de Dios* neighborhood.

2. Basic Concepts

2.1. Slums

2.1.1. Definition and Properties

Despite that in the literature, there are several ways to define slums, the one that is most useful in the context of this work is that a slum is a set of buildings of spontaneous origin in a landscape. Due to its spontaneous origin, it turns out that the slums seem to be planless or antiplan[11]. An attempt to classify the slums is shown in [11], where they indicate that there are two key factors to determine the type of slum: hope (or despair) and escalator (or non-escalator), see Figure 1.

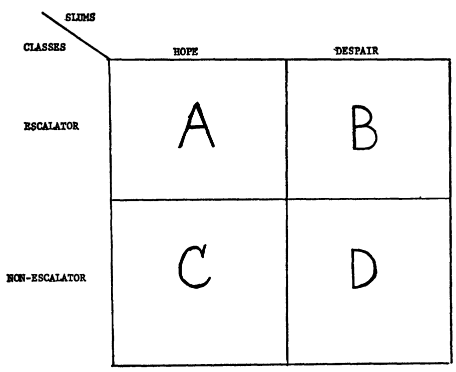

Figure 1: Slums classification. Reprinted from "A theory of slums," by C. J. Stokes, 1962, Land economics, vol. 38, p. 189. Copyright Year by JSTOR.

The hope factor is related to the reason for the settlement, whether it is done by necessity or in search of better opportunities. While the escalator factor is related to the integration of slum dwellers for reasons of culture, religion or race.

The slums are categorized as types A, B, C, D. In practical terms, most of type A slums are the result of migrations of people from the same country who have some economic capacity. While the slums belonging to the other types are formed by people of very different origins or with no economic capacity. This causes that differences can be appreciated in the ordering of the constructions according to the type of slum. For instance, in the slums of type A we could find a kind of linear arrangement of the constructions because people living there have the intention to stay there for a while, but in slums of type D the constructions seem to be more random because their dwellers were probably settling for need, see Figures 2 and 3.

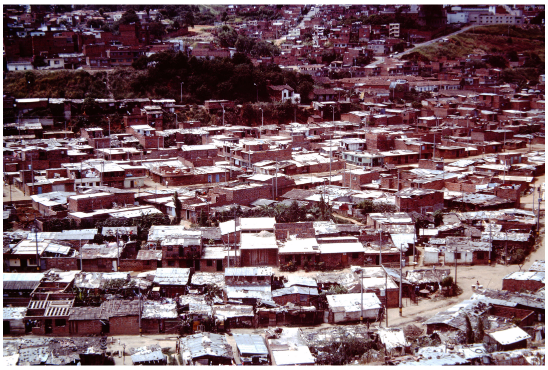

Figure 2: Slum of type D. By Institute for Housing and Urban Development Studies, CC BY-SA 3.0, https://commons.wikimedia.org /w/index.php?curid=34389333

Figure 3: Slum of type A. Reprinted from “SLUM ALMANAC 2015/2016: Tracking Improvement in the Lives of Slum Dwellers,” by UN-Habitat, 2016, Participatory Slum Upgrading Programme, p. 9. Copyright Year by UN-Habitat/Julius Mwelu.

It is important to take into account that since the proposed factors for the classification are qualitative, a slum could be of type A but still have certain similarities with those of other types.

2.1.2. Slums Upgrading

According to the UN-Habitat, the policies used to deal with slums in the last decades can be grouped into three categories: laissez-faire, restrictive or preventive, and supportive[6].

The laissez-faire strategies (let do strategies) part of the fact that the slums are a temporary phenomenon, that is, the inhabitants of them are positioned there until they improve their economic situation and then move to another more appropriate place. However, for the majority of slum dwellers, this was not achieved, because the costs of living in other places are prohibitive for them, or because moving to other places they would be distant from possible sources of employment[6].

The second type of strategies were based mainly on the resettlement of slum dwellers to state public housing. The problem with this was that the number of people living in slums is incredibly high, so resetting them involves a huge expense[6]. It has also been found that although resettled people improve their living capacity, many of them prefer to return to their former settlements in slums[4].

The last type of strategy uses a participatory approach from the public sector to improve the conditions of current settlements. For these strategies to be the most effective, it is desirable to involve the inhabitants in the planning of the policies in order to take into account the peculiarities of the site, among other things[6]. According to the UN-Habitat, intervening in slums for a street-based strategy is one of the best ways to improve slums. This is mainly because in this way tangible results can be seen, such as improvement in accessibility, infrastructure, design layout and legalization of land tenure[6].

In [1], a street-based strategy for the improvement of slums using topological algorithms is proposed. The urbanist Bertaud is a detractor of the use of strategies guided by topological algorithms to restructure the slums. Because usually, the solutions that involve the construction of roads need to create space, which will be obtained through the eviction of some inhabitants[8]. However, the reality of slums in Ecuador is that they are not as dense as those that can be found in other countries such as India for example. This makes it easier to find ways to create these roads without moving the current buildings.

2.1.3. Slums in Guayaquil

According to Camila Mackliff, 59% of Guayaquil's surface is occupied informally [12]. This is due to the fact that the growth of the city took place through the informal accumulation of people who migrated to the city, at first, fleeing the adverse conditions generated by the economic crises that occurred in the history of Ecuador but now to take advantage of the economic conditions provided by the city with the largest maritime port in Ecuador[13].

Given the current conditions of creation of the slums, it can be said that most of the new slums in Guayaquil are of type A. In particular, the *Ciudad de Dios* neighborhood which in 2015 was an area with few numbers of houses but now can be found large spontaneous constructions, see Figure 4. An important characteristic of the informal settlements of Guayaquil is that normally the lands where they are located belong to land traffickers, which means that the location of the houses is carried out in lots and that it is not completely random[13].

Figure 4: Satellite image of *Ciudad de Dios* in 2015 (left). Satellite image of *Ciudad de Dios* in 2018 (right).

2.2. Topological Graph Theory

Before showing the results of the study of the Topology of the Cities, it is necessary to review some general concepts of the theory of topological graphs. Our principal resources are [14], [15], [16] and [17].

Definition 1 (Graph)**.** *A graph G is a pair of sets (V, E), where V is a finite non-empty set of elements called vertices, and E is a finite set of elements called edges, each of which has two associated vertices (which may be the same).*

The sets V and E are the vertex-set and edge-set of G and are sometimes denoted by $V(G)$ and $E(G)$. The order of G is the number of vertices, usually denoted by n, and the number of edges is denoted by m, see Figure 5.

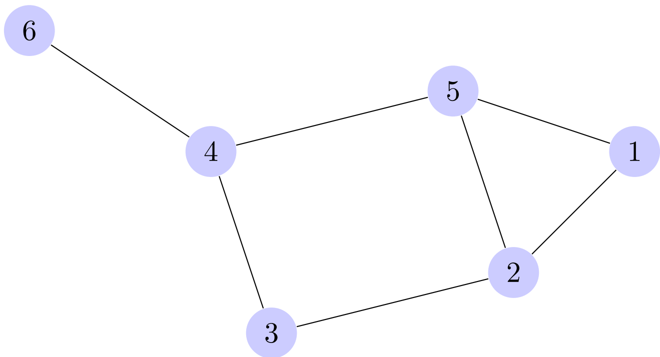

Figure 5: Graph of order 6.

Definition 2 (Walk)**.** *A walk in a graph is a sequence of vertices and edges $v_0, e_1, v_1, \ldots, e_k, v_k$, in which each edge e_i joins the vertices v_{i-1} and v_i. This walk goes from v_0 to v_k or connects v_0 and v_k, and is called a $v_0 - v_k$ walk.*

- a path is a walk in which no vertex is repeated;
- a cycle is a non-trivial closed walk in which no vertex is repeated, except the first and last;
- a trail is a walk in which no edge is repeated;
- a circuit is a non-trivial closed trail.

Definition 3 (Surface)**.** *A (topological) surface is a topological space in which every point has an open neighborhood homeomorphic to some open subset of the Euclidean plane E_2.*

Definition 4 (Genus of a Graph)**.** *The genus $\gamma(G)$ of a graph G is the minimum genus of a surface in which the graph can be embedded – that is, the minimum number of handles that need to be added to the sphere for G to be embeddable.*

A connected graph of genus 0 is said to be planar. In general, a graph is planar if all of its components are planar. Thus, non-empty connected planar graphs are the graphs embeddable in the sphere. They are the ones that underlie spherical gems. In general, the determination of the genus of a graph is an unsolved problem.

There are two kinds of closed surfaces, orientable and nonorientable. A surface is *orientable* if a positive sense of rotation (say, clockwise) can be made around all points consistently, and is $non-orientable$ otherwise. The sphere, the torus, the double torus, the triple torus, and so on, are orientable. They are commonly denoted S_0, S_1, S_2, S_3, ... Moreover, every closed connected orientable surface is homeomorphic to one of them. Another characterization of the closed orientable surfaces is that each one can be obtained by adding some handles to a sphere. Adding one handle yields S1, adding two yields S2, and so on. See Figure 6.

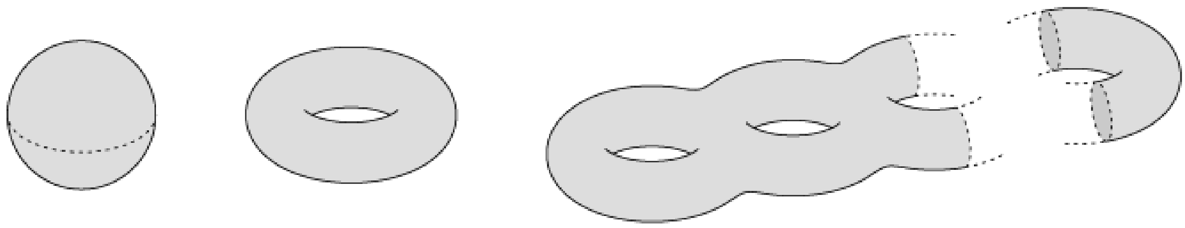

Figure 6: A 2-sphere (genus 0), a torus (genus 1) and an orientable surface of higher genus. By Daniel Müllner, http://www.map.mpim-bonn.mpg.de/images/5/57/Surfaces.png

A sphere and a torus are examples of closed surfaces. They have no punctures, they do not run off to infinity, and they do not have any sharp boundaries. Sometimes we want to consider surfaces that are not closed. A disk and a cylinder are examples of surfaces with boundary. A surface with boundary is still locally 2-dimensional, except that it may have one or more 1-dimensional boundary curves, see Figure 7. The surfaces of Figure 7 are equivalent to each other. This is because one can continuously deform one to obtain the other.

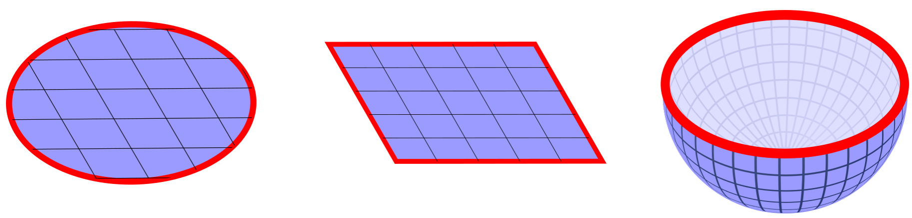

Figure 7: Some examples of orientable surfaces with boundary. By File: Simple Torus.svg: YassineMrabet File: Sphere wireframe 10deg 6r.svg: Geek3derivative work S by rnes321 - File: Simple Torus.svg File: Sphere wireframe 10deg 6r.svg, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=12445003

A very important property that relates graphs with surfaces is that of "embedding", which is related to the notion of being able to draw a graph on a surface without the arcs of it crossing each other.

Definition 5 (Embedding of a Graph on a Surface)**.** *An embedding of a graph G on a surface S is a one-to-one mapping of the vertices of G into that surface and a mapping of the edges of G to disjoint simple open arcs, so that the image of each edge joins the images of its two vertices and none of the images of the edges contains the image of a vertex.*

The plane is not closed, but since it differs from the sphere by only a single point, it follows that a given graph can be embedded in the plane if and only if it can be embedded in the sphere.

A *region* of an embedded graph G is a maximal connected set of points in the relative complement of G in the surface; note that one region is unbounded. The topological closure of a region (that is, the region together with the vertices and edges of G on its boundary) is a *face*.

Definition 6 (Cellular Embedding). *An embedding is cellular if every region is homeomorphic to an open disc.*

Theorem 1 (Euler's formula). *If a simple graph G has a cellular embedding in a surface S with n vertices, m edges and r regions, then $n - m + r = 2 - 2h$, where h is the genus of the surface S_h.*

The number associated with S in this theorem is called its Euler characteristic.

Definition 7 (Poincaré Dual Embedding). *The* Poincaré dual embedding *for a cellular graph embedding $G \to S$ (called* the primal embedding *in this context) is constructed as follows:*

- *in the interior of each primal region, a* dual vertex *is drawn;*
- *through each primal edge, a* dual edge *is drawn joining the dual vertex on one side of the edge to the dual vertex on the other (thus, a loop whenever the same primal region lies on both sides of that primal edge);*
- *if the surface S is oriented, then in the dual embedding, the orientation is reversed.*

The dual graph has been used to perform several proofs in which the initial problem was not established on graphs. For example, in the demonstration of the four-color map theorem. The theorem indicates that in a map of a country with continuous regions, it is only necessary at most four colors so that two continuous regions are not colored the same. In terms of the dual graph, this problem is reduced in that two adjacent nodes do not have the same color, see Figure 8.

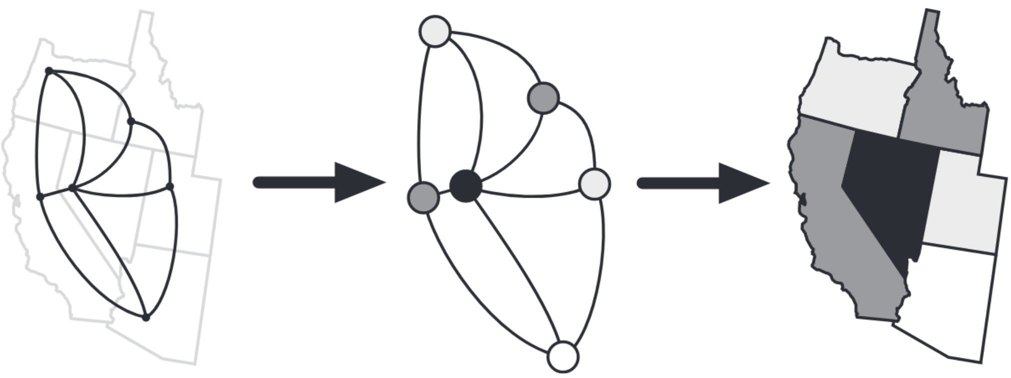

Figure 8: Coloring the adjacency graph gives a coloring of the map. Reprinted from "Euler's Gem: The Polyhedron Formula and the Birth of Topology," by David S. Richeson, 2008, Princeton University Press, p. 136. Copyright Year by Princeton University Press.

A famous graph is the Birkhoff diamond, that it is used to study the colorability of some graphs[18], this graph is shown in Figure 9. Particularly, George D. Birkhoff states that the configuration of that graph cannot be present in a minimum counterexample to the four color theorem[19].

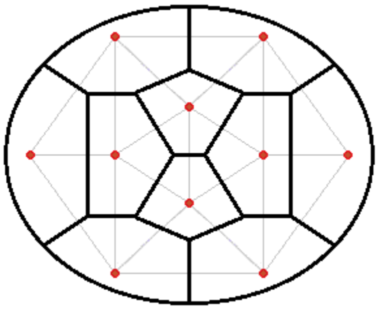

Figure 9: Birkhoff diamond. By Snorri95, CC BY-SA 3.0, https: //commons.wikimedia.org/ w/ index.php?curid=24977673

2.3. Topological Optimization

The size, shape, and topology have a big importance in the field of structural design engineering since these design features have a high-impact on the performance of the final product[20].

The problems of size, shape and topology optimization are resolved in different ways. Particularly, the topological optimization of structures is involved in the determination of characteristics such as the number, location, and shapes of the holes that will have the design[20]. This translates into establishing in which points of space should have had material and in what points should remain empty. For instance, when topological optimization is implemented in the design of a car wheel different feasible solutions could be obtained according to the number of holes that must be added in the design region[21]. One of these solutions is showed in Figure 10.

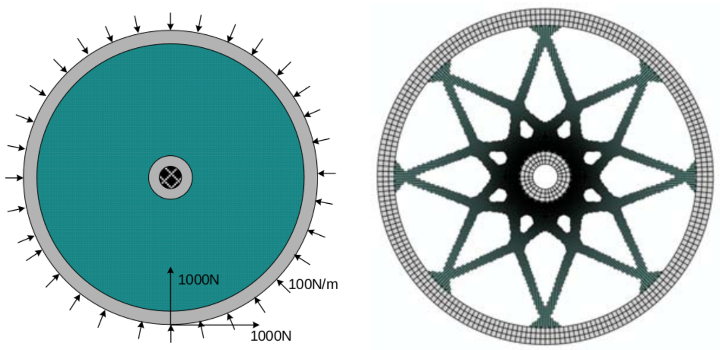

Figure 10: Topological optimization for a wheel under different loads. Reprinted from "Topology Optimization of Periodic Structures," by Zhihao Zuo, 2009, RMIT University, p. 87. Copyright Year by RMIT University.

The result obtained by using topological optimization can lead to new and innovative designs[22]. And these designs could serve as a starting point for then use other optimization criteria of interest to the designer[20].

There are several methods to perform topological optimization, one of the most common is the Evolutionary Structural Optimization Method (ESO). This method is based on removing unnecessary material from the structure and thus evolving to the optimal shape and topology of the object[23]. If we divide the fixed design domain in grids, it would be as pixels for an image, then the solution of a topological optimization through ESO could be understood as removing pixels from an image, see Figure 11.

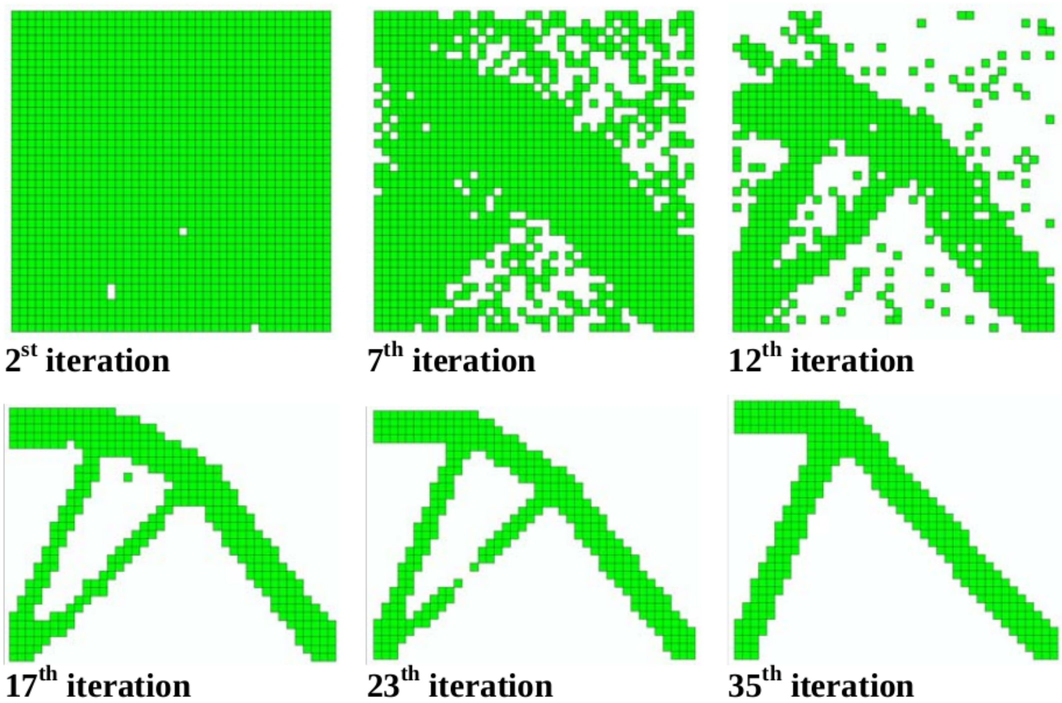

Figure 11: Intermediate half designs of a topological optimization example. Reprinted from "Topology Optimization of Periodic Structures," by Zhihao Zuo, 2009, RMIT University, p. 208. Copyright Year by RMIT University.

Besides the use of topological optimization in the field of structural design, this type of optimization is also used to obtain more robust communication networks that are governed by different design restrictions and have the lowest possible cost[24]. For example, it is possible to use a heuristic approach to be able to determine the topology of an optimal Internet network. Starting from a shorter tree (minimum expansion tree) and then adding connections between nodes that have a lot of data traffic one could have a robust internet network[24].

3. Results

In order to achieve the goals of this project, we proceed in the following way. First, we carry out a bibliographic review of the relevant definitions about slums as their characteristics and classification. Then, an analysis of the situation of the slums in the city of Guayaquil was made, with emphasis on the *Ciudad de Dios* sector. Subsequently, the bibliographic review was aimed at compiling the concepts of topological graph theory that we are going to use in this work. Then, we follow the approach proposed in [1]. The main goal here was to understand how the tools of topology and graph theory can be used to diagnose and solve neighborhood development problems. The next step was to reuse and modify the tools that the authors of [1] made available to the public for the implementation of their approach. After that, an academic example was developed to be able to obtain some different solutions that the approach can give us. Finally, a sector of the *Ciudad de Dios* neighborhood was digitized to execute the approach and analyze the result that it has produced when being used in an Ecuadorian slum.

In this section, we introduce some topological concepts that are used to demonstrate that using the topological properties of the neighborhoods is a valid methodology to characterize them.

3.1. Topology of a City

The topology of cities/neighborhoods can be understood as the topological relationship between the two main components of the city: its streets and its buildings. This is the reason why it is necessary to understand the topology of the access systems and the topology of the constructions, to then see how they are related according to each city.

3.1.1. Topology of the Access System

The set of paths and roads of any city is called the urban access system. The access system has the property of being **path-connected**, i.e., any two points on this surface can be connected traveling on the surface. Moreover, access systems are **orientable**, **2-dimensional surfaces** and **compact**. These properties are related with the following facts: there is a global definition of up from down, we can only move on road and path surfaces and the surface is finite.

Moreover, the urban access system is a 2D surface with both **internal** and **external boundaries**: the limit of the city is the external boundary, while boundaries between accesses and each place are the internal boundaries. So, for a city with b blocks, there are b internal boundaries and one external boundary so that the total number of boundary components, B, is $B = b + 1$. In this sense, a city subsection is defined as a set of contiguous city blocks and surrounding access system, including an external boundary that defines the physical limits of the subsection. When a subsection includes all blocks it is equivalent to the city and shares its entire access system.

The authors in [1] show that the topology of the access system of a city with b blocks is equivalent to the topology of a sphere with $b + 1$ disks removed. In consequence, urban access systems are topologically equivalent if, and only if, they have the same number of blocks. Moreover, if we use a graph representation of the urban access system (Y), as the one suggested in [25], where edges correspond to roads or paths, and nodes correspond to their intersections, Y has the same Euler characteristic as the urban access system, $\chi(Y) = 1 - b$.

These deductions are summarized in the following theorem and corollaries which were obtained from [26]:

Theorem 2 (Topological Classes of Urban Access System)**.** *The access system of any city with b blocks is topologically equivalent to a sphere with $B = b + 1$ disks removed.*

Proof. The proof of this theorem is based on the fact that the surfaces with boundaries can be constructed by removing open discs from the surfaces without boundary. Therefore, as the access system is orientable, 2-dimension and has genus zero; it is topologically equivalent to a sphere but with b+1 open discs removed. □

The proofs of the following four corollaries are trivial by the definition of topological equivalence and the previous theorem. So, we include only the proof of the fifth corollary.

Corollary 2.1. *For any value of b, two cities with b blocks have access systems that are topologically equivalent, since both cities' access systems are topologically equivalent to a sphere with $b+1$ disks removed.*

Corollary 2.2. *Any subsection of one city with b blocks has an access system that is topologically equivalent to that of another subsection of another city with b' blocks, if and only if $b = b'$.*

Corollary 2.3. *The access system of an entire city with b blocks is topologically equivalent to a subsection of any other city with the same number of blocks.*

Corollary 2.4. *Urban Access Systems are topologically equivalent if and only if they have the same number of blocks.*

Corollary 2.5. *A 1-complex graph representation of an urban access system with b blocks, where edges correspond to road and path centerlines and nodes correspond to their intersections, called the urban access network, Y, has the same Euler characteristic as the urban access system, $\chi(Y) = 1 - b$.*

We recall the proof of this corollary due to Brelsford et al. in [26].

Proof. Let's compute the Euler characteristic of a general surface S with B boundaries.

First define a surface C, where S^* corresponds to S with all the B boundaries patched by disks.

Then, $\chi(S^*) = \chi(S) + B * \chi(Disk)$. Since, $\chi(Disk) = 1$ and $\chi(S^*) = 2$ because S^* is a sphere.

$\chi(S) = 2 - (1 + b) = 1 - b$.

On the other hand, a 2-complex planar graph has $\chi = v - e + f = 2$, while a 1-complex planar graph has $\chi = v - e$. Let be Y_2 a 2-complex graph representation of the urban access system, then, Y_2 has one face per each boundary so $f = b + 1$.

Then, $\chi(Y_2) = v - e + f = 2$
$v - e = 2 - f = 1 - b = \chi(Y)$. Therefore, $\chi(Y) = \chi(S) = 1 - b$ □

3.1.2. Topology of City Blocks

Brelsford et al. use the general term parcel in [1] to denote the decomposition of the city block land area into separate units: these are buildings, or more generally, separate land holdings and include public places that are not accesses. When a parcel is adjacent to the access network. it is said that this parcel is accessible. In the case where the parcel is not adjacent to the access network. it is internal to the block, this implies that its access is mediated through other parcels. However, interior parcels can be connected to the urban access system by converting edges in the S_0 graph from parcel boundaries to roads, where the edges of S_0 correspond to the boundaries of each parcel and its nodes to their intersections, see Figure 12.

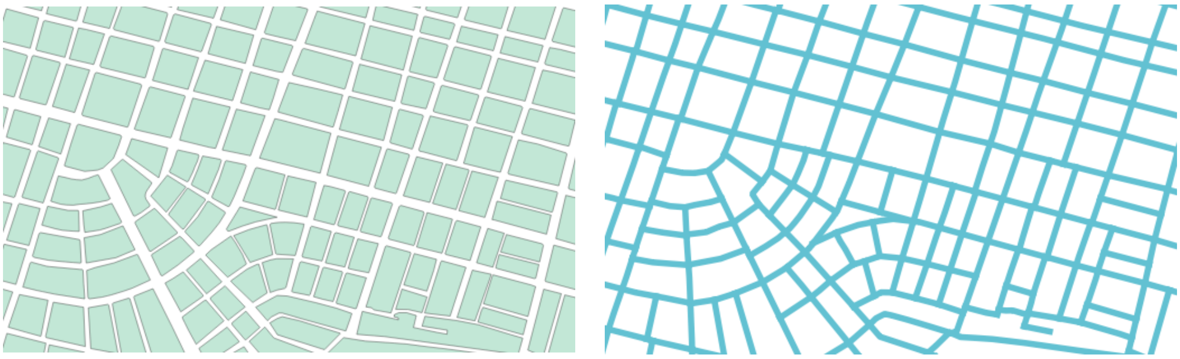

Figure 12: Parcels of a neighborhood in Guayaquil (left). Corresponding S_0 graph (right).

One characteristic of most of the planned neighborhoods is that all their parcels are accessible whereas slums are more likely to have inaccessible parcels [6]. Then, the connectivity of the neighborhoods is a feature to take into account. This characteristic of the neighborhood can be analyzed using a metric k_{max}, called block complexity, which measures the connectivity of a city block. Additionally, they have shown that it is possible to find k_{max} by iterating the construction of weak dual graphs to S_0; the weak dual graph of S_0 is S_1, where each parcel becomes a node and adjacency becomes an edge. This procedure should be done iteratively ($S_{k-1} \longrightarrow S_k$) until there are not more regions in the graph S_k. Moreover, they found that the complexity of universally accessible blocks (blocks with all parcels accessible) is $k \leq 2$, while non-universally accessible blocks (blocks with interior parcels) will have $k > 2$. Furthermore, a block S is universally accessible if, and only if, S_2 is a tree. And, if a parcel is represented with a node in the S_k graph, at least $\frac{k-1}{2}$ parcel boundaries must be crossed in order to reach it from the nearest section of the access system.

Finally, in the minimal set of additional roads necessary to have a universal accessible block, there will be no loops. Thus, newly constructed roads in the minimal set of accesses form a tree or set of trees (culs-de-sac). This feature is interesting because loopless configurations have seemed in river basins where every spanning tree is exactly a local minimum of total energy dissipation [27].

These statements are shown in more detail, as in [26], in the following.

Definition 8. *A block S is called universally accessible if every parcel within S adjoins a road. Otherwise, S is not universally accessible.*

Definition 9 (minimal set of accesses)**.** *Interior parcels can be connected to the urban access system by converting edges in the S_0 graph from parcel boundaries to roads. The minimal set of additional roads necessary to connect a given parcel to the road system is the set of edges with the shortest total length such that at least one node contained in the set of edges to be converted is part of an existing road, and at least one node is part of the face in S_0 that surrounds the parcel.*

Definition 10. *In a graph G, a cycle is a collection of m vertices and m edges arranged so that each vertex has exactly two edges incident to it, where $m \geq 3$.*

Definition 11. *A face of a planar graph is a maximal region in the plane that contains no edge or vertex of the graph.*

Definition 12 (Weak Dual Graphs). *For each bounded face of S_0, we assign a vertex in S_1. Two vertices of S_1 have an edge between them if and only if the faces of S_0 they represent share a common border of at least one edge in S_0. Then, S_1 is the weak dual graph of S_0. For a block S, we may then assign a stage k graph, S_k, defined recursively by repeating the process used to construct S_1 from S_0 on the stage $k-1$ graph S_{k-1}.*

In Figures 13, 14 and 15, there are shown the correspondent construction of the successive stage graphs. Moreover, the original graph has a block complexity of 4.

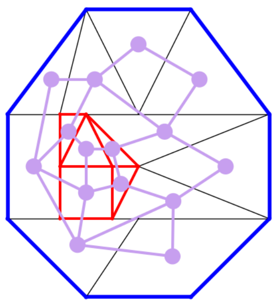

Figure 13: Stage graph S_1

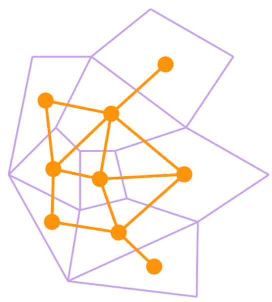

Figure 14: Stage graph S_2

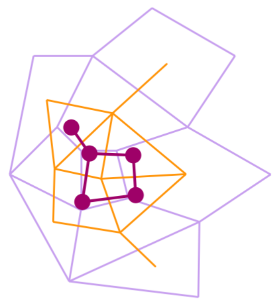

Figure 15: Stage graph S_3

Definition 13 (block complexity). *The complexity of the block S is the smallest positive integer k such that S_k is a tree. Every block will be characterized by a positive, discrete value of this complexity.*
The complexity of universally accessible blocks is $k \leq 2$. Non-universally accessible blocks will have $k > 2$.

Definition 14. *A graph G is called a tree if G contains no cycles.*

Definition 15. *A vertex v of a graph G is called an interior vertex if there exists a cycle surrounding v so that deleting this cycle from G results in either:*

1. *Two connected components, one of which contains vertex v and all of its incident edges, or*

2. *Just the vertex v and its incident edges.*

Theorem 3. *A block S is universally accessible if, and only if, its stage-two graph, S_2, is a tree.*

We now bring the proof that Brelsford et al. developed for this theorem in [26].

Proof. Let's assume that S_2 is not a tree. This means that there exists an interior face of S_2 whose boundary is a cycle σ consisting of m vertices$x_1, x_2, ..., x_m$ of S_2 and m edges. Each vertex x_i in σ represents a face f_i of S_1, where face f_i shares a common edge with face f_{i-1}(mod m)and face f_{i+1}(mod m). Furthermore, each of these shared edges is incident to a vertex v of S_1 that represents the interior face of S_2. Thus, the cycle σ in S_2 corresponds to a subgraph of S_1 consisting of the m faces, $f_1, f_2, ..., f_m$ arranged in a circle around the vertex v. This means that vertex v is an interior vertex of S_1, so it corresponds to a parcel of the block S that does not border a road. This shows that block S is not universally accessible. Now, we will prove that if a block S is not universally accessible, its stage two graph, S_2, is not a tree. We assume that there exists a parcel n of a block S that does not border a road. Thus, there is a vertex

v_n of S_1 corresponding to parcel n that is an interior vertex of S_1. Consider the subgraph V_1 of S_1 consisting of a minimal cycle surrounding vertex v_n, vertex v_n itself and all edges incident to vertex v_n. Now, we consider the subgraph V_2 of S_2 that represents V_1. V_2 will contain one vertex for each face of V_1 connected by one edge representing each edge incident to vertex v_n. We conclude that the subgraph V_2 of S_2 is a cycle with m vertices, where m is the degree of vertex v_n in S_1. This says that the stage two graph of S contains a cycle, and is therefore not a tree. □

Theorem 4. *If a parcel is represented with a node in the S_k graph, at least $\frac{k-1}{2}$ parcel boundaries must be crossed in order to reach it from the nearest section of the access system.*

We recall the following proof due to Brelsford et al. in [26].

Proof. For any parcel n of block S, the minimum number of parcel boundaries that must be crossed to reach a road is represented by the minimum number of edges necessary to form a path from v_n, the vertex representing n in the S_1 graph, to an exterior vertex of S_1.

Observe that, in the algorithm for creating the S_k graph of a block S, parcels of S are represented by faces of S_k when k is even and nodes of S_k when k is odd. Furthermore, for even k, if a face of S_k touches an exterior vertex, that face is represented by an exterior vertex in S_{k+1}. Finally, observe that, for odd k, the parcels represented by an exterior vertex of S_k are not represented at all in S_{k+1}.

Therefore, suppose a parcel n requires a path of length l to connect vertex v_n to an exterior vertex in S_1. It is clear that in S_3, the path from v_n to an exterior vertex will have length $l-1$, and so on. The vertex v_n will thus be an exterior vertex of the graph S_{1+2l}. Therefore, we see that, if vertex v_n appears in graph S_k, then $k \leq 1+2l$, which says that $\frac{k-1}{2} \leq l$. □

Theorem 5. *There will be no loops in the minimal set of additional roads necessary to connect all interior parcels to a road. Thus, newly constructed roads in the minimal set of accesses form a tree or set of trees (culs-de-sac).*

We now present the proof effectuated by Brelsford et al. in [26].

Proof. We may consider the access network of a given block as a subgraph of the stage zero graph $S0$. To connect all parcels to a road, we consider parcel boundaries, which are represented by interior edges in $S0$. We may then choose a set of such edges of $S0$ to represent additional segments of road needed to ensure that the block is universally accessible. There will be several choices for this set of additional roads; we choose the one that has the fewest total geometric length of edges (minimal set of accesses).

Suppose that there exists a block for which the minimal set of additional roads is not a tree or set of trees. Let M denote the subgraph of $S0$ consisting of edges belonging to the minimal set of roads along with the nodes incident to these edges. We are assuming that there is at least one cycle in M. Every face of $S0$ representing an interior parcel must share at least one node with M in order for every parcel to be accessible via existing or new paths. However, all connected planar graphs have a spanning tree, which is a subgraph containing all nodes of the graph but no cycles. Then, we let M' be the subgraph of M consisting of spanning trees for each component of M. Thus, every face of $S0$ representing an interior parcel will share a node with M', making every parcel accessible via existing or new roads, but M' has strictly fewer edges than M, as it is a subgraph containing no cycles. This contradicts the choice of M as minimal. Therefore, the set of newly constructed roads must form a tree or set of trees □

3.2. Minimal Neighborhood Reblocking

In [1], the authors describe two ways to analyze the related topological optimization problem. The first, a strict optimization that is too rigid for practical use. The second, a statistical optimization that is more flexible and can be the basis for practical neighborhood upgrading tools.

The strict optimization approach is based on finding the smallest possible configuration of the streets to ensure that each interior parcel is connected to the access system. Although this strategy finds the best solution, the computational complexity grows in a combinatorial way according to the number of parcels that are considered. This is because there may be solutions with shorter lengths if we consider a pair, trio, etc. of internal plots together. Thus, this approach is not practical for real situations.

The statistical optimization uses the fact that some paths, despite not being the most optimal, have important features. Then, this technique allows using additional data to the problem of strict optimization, such as the price of building new streets or how straight the roads are. As the criterion of the cost of construction of streets is a function of the street length, it is understandable why this strategy of optimization delivers as a result that the smaller roads must be built more likely.

The approach developed by Brelsford et al. to deal with blocks within a large number of parcels consists of the following steps. First, find the solution to strict optimization for an interior parcel. Then, define a set of feasible solutions adding n alternative paths to connect the parcel with the access system. This process must be repeated for the rest of the interior parcels. Then, use the defined probability function for statistical optimization to select a single path from our set of feasible path solutions. After adding the selected path to the access system, the information of the interior parcels is updated and the whole process is repeated until there are no more interior parcels, i. e., the graph S_2 is a tree.

4. Implementation

The approach was implemented and tested using two examples: an academic one and a real-life one. The real-life implementation was developed using a zone of the neighborhood *Ciudad de Dios* in Guayaquil.

The objective to use an academic example is to recreate the results proposed in [1], and in this way to be able to verify the different solutions that the algorithm gives us depending on the type of optimization used, i.e., strict optimization(piecewise) or statistical optimization. Moreover, we were able to validate the construction of the dual graph for a block with interior parcels as the theory related to it.

On the other hand, the motivation to use a real-life example is to show that this proposal is viable to be used in the context of the Ecuadorian slums.

4.1. An Academic Example

The academic example consists of 15 parcels, where 10 of them are exterior parcels and five are interior parcels, see Figure 16.

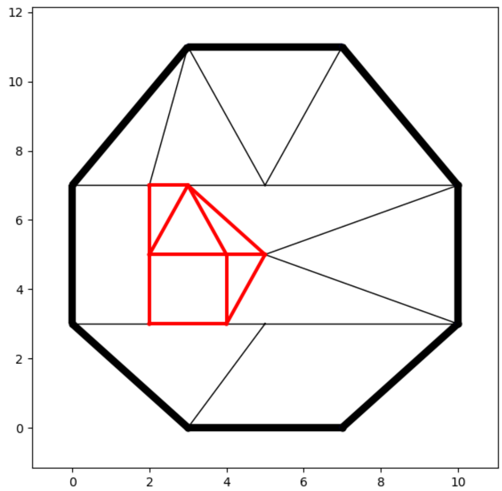

Figure 16: Academic example. Red lines correspond to interior parcels limits.

From this example is possible to realize that the corresponding stage graph S_2 (see Figure 17) has a loop, which means that our block is not universally accessible. Therefore, we can use the approach in order to upgrade his interior parcels to external parcels.

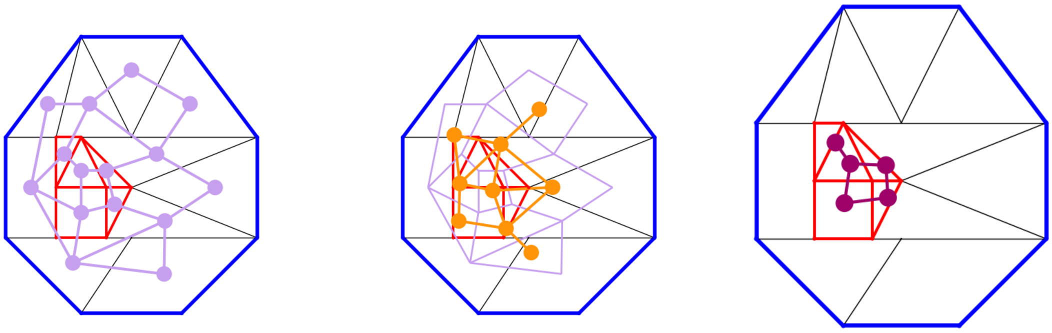

Figure 17: From left to right: Stage graphs S_1, S_2, and S_3.

Using the strict optimal (piecewise) algorithm in our academic example, the solution is constructed adding the smallest road in order to change an interior parcel into an exterior parcel in each iteration, see Figure 18. A total of four iterations was necessary in order to have a universally accessible graph.

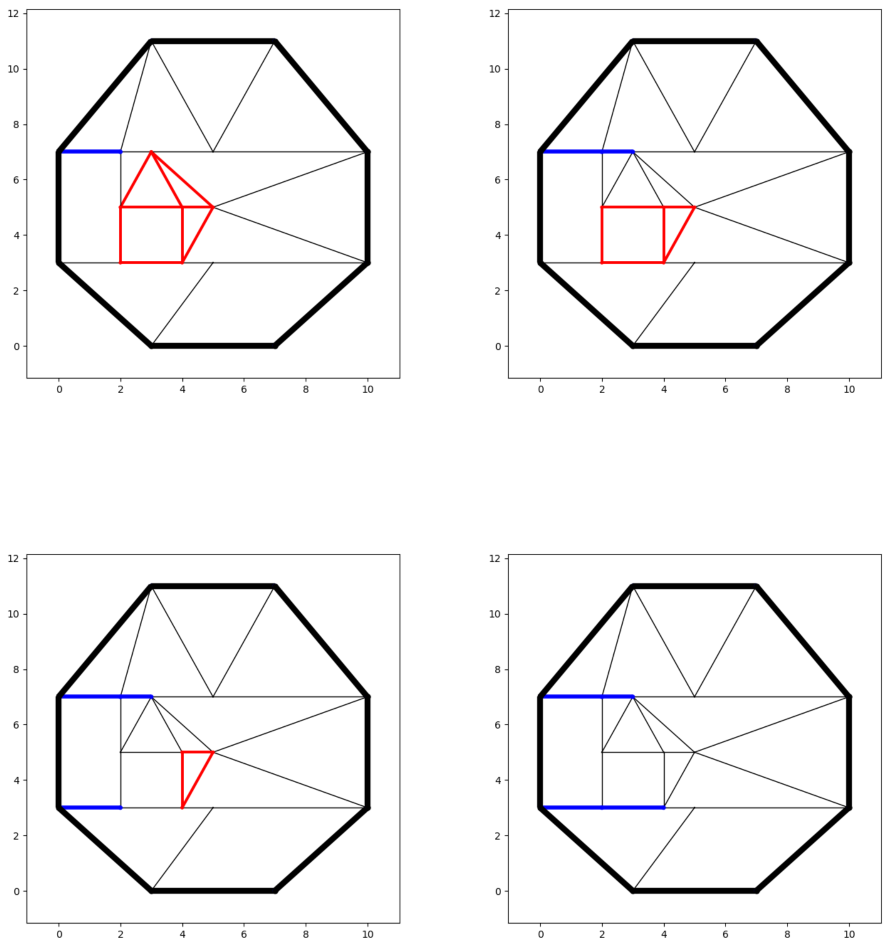

Figure 18: Step by Step execution of the strict optimal approach.

In Figure 19, we can see the dual graphs S_1 and S_2, respectively. Since S_2 is a tree the complexity of this block is 2. Therefore, this block is universally accessible.

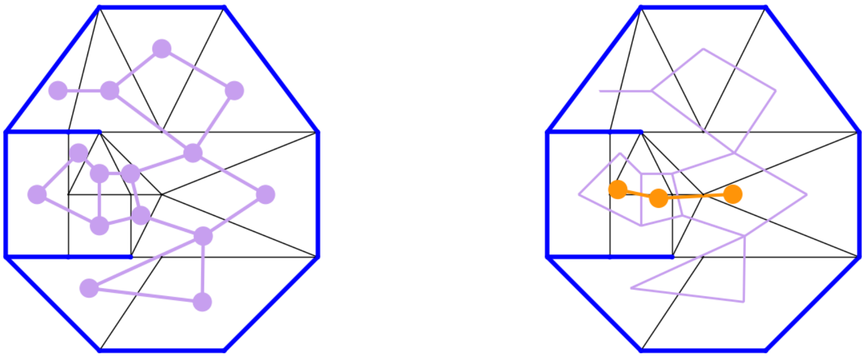

Figure 19: Stage graphs S_1 and S_2 for the solution of strict optimization.

Following, two solutions obtained using the statistical optimization approach are shown in Figures 20 and 21. According to the scale used, one solution path has around of 7.82 meters of new streets, while the other solution has around of 6.83 meters of new roads. Since the optimal path solution computed with the strict optimization (piecewise) strategic has a length of 7 meters of new streets, see Figure 16. This shows that the solution constructed using strict optimization (piecewise) is not always the globally optimum.

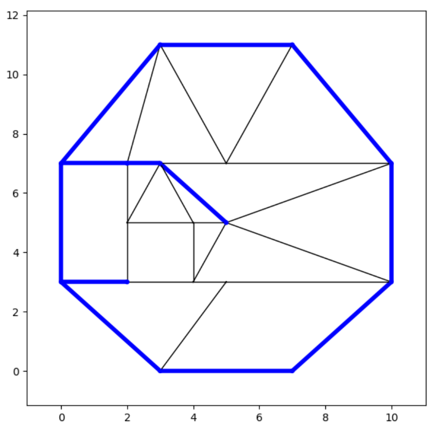

Figure 20: Solution 1 from executing statistical optimization

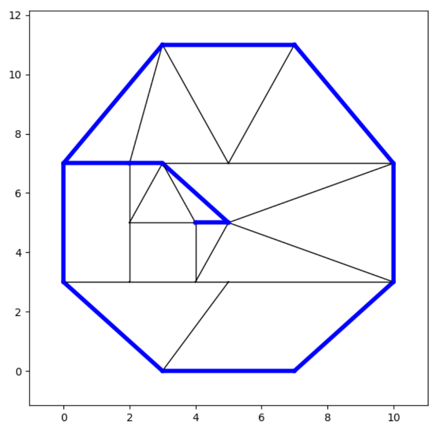

Figure 21: Solution 2 from executing statistical optimization

4.2. *Ciudad de Dios* neighborhood in Guayaquil

The real life application was developed in a sector of *Ciudad de Dios* in Guayaquil, see 4. This sector has about 377 parcels of which, 281 are interior parcels, see Figure 22.

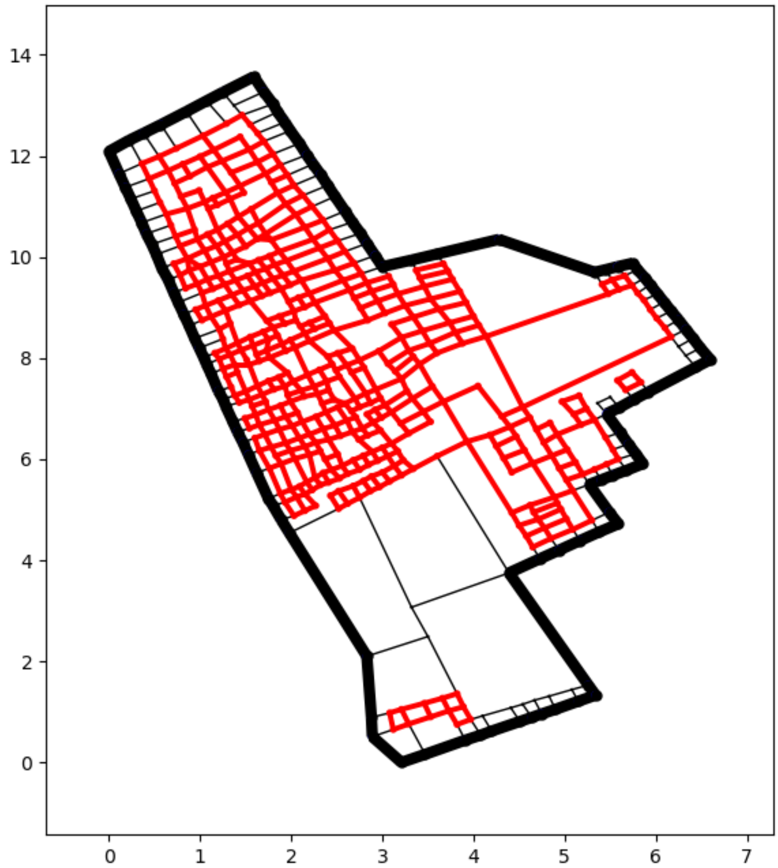

Figure 22: Graph representation of Ciudad de Dios's sector. Red lines correspond to interior parcels limits.

The result of executing the statistical optimization gives us a block that is universally accessible. Therefore, it is topological equivalent to a planned neighborhood, see Figure 23. This solution has a lot of culs-de-sac this was expected from the fact that this configuration gives us an instance of a minimal set of streets needed to have every parcel connected to the connection system.

As we saw in the Section 2, the topological optimization serves as a starting point to then modify the solution so that it meets other desired criteria. These criteria may be to decrease the geometric distance between pairs of plots or to make the new roads as straight as possible in order to facilitate the delivery of other basic services such as street lighting.

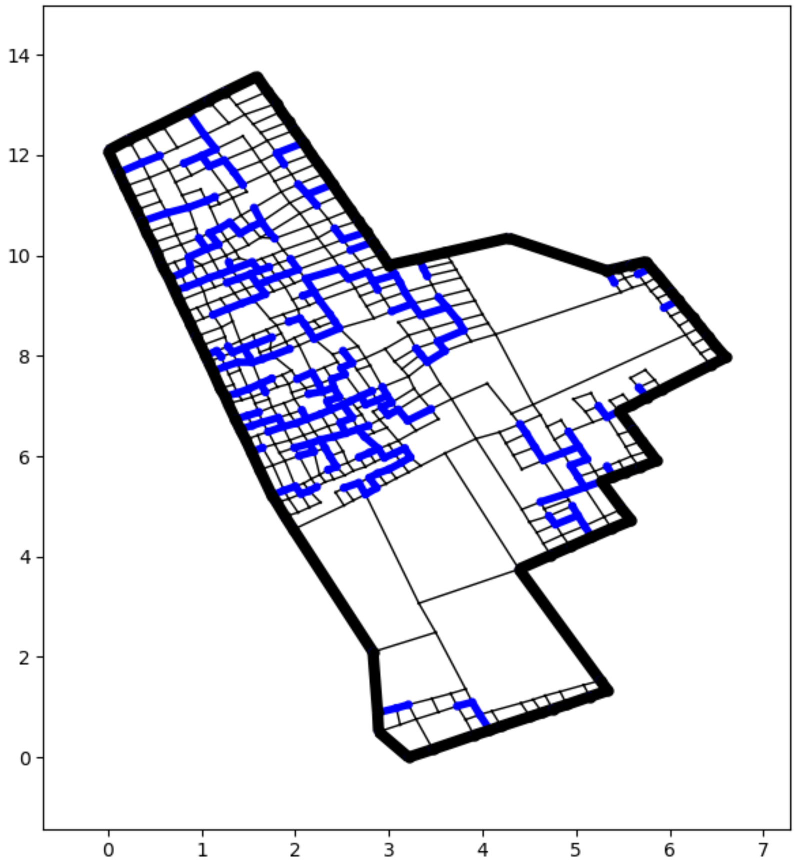

Figure 23: Graph solution for executing the statistical approach in the sector of Ciudad de Dios.

5. Conclusions

Analyzing neighborhoods and cities through topological tools and graph theory allows to characterize slums with respect to planned neighborhoods. It was found that the planned neighborhoods are universally accessible, i. e., all their parcels are connected to the network access system. While slums are not universally accessible. This characteristic can be determined using a topological tool known as weak dual graphs; it was found that a block is universally accessible, if and only if, his corresponding stage-two graph S_2 is a tree. Which allows to develop an algorithmic solution of the re-blocking problem with optimal cost as done in[1]. Since executing an optimization algorithm could be a very time-consuming process, a statistical optimization algorithm was applied. The advantage of this approach is that we can add more information about the re-blocking problem and instead of having the global optimal solution we could have a more suitable solution to be implemented in the slum.

Given the characteristics of most of the slums in Ecuador, we firmly believe that implementing this mathematical approach in Ecuador could improve the situation of many cities in our country.

References

[1] C. Brelsford, T. Martin, J. Hand, and L. M. A. Bettencourt, "Toward cities without slums: Topology and the spatial evolution of neighborhoods," *Science Advances*, vol. 4, no. 8, 2018.

[2] C. T. I. para el proceso preparatorio de Habitat III, "Posición nacional frente a habitat iii," Ministerio de Desarrollo Urbano y Vivienda, Tech. Rep., 2018.

[3] *Ley Orgánica de Ordenamiento Territorial, Uso y Gestión de Suelo*, Asamblea Nacional del Ecuador, 2016. [Online]. Available: https://www.habitatyvivienda.gob.ec/wp-content/uploads/downloads/2016/08/Ley-Organica-de-Ordenamiento-Territorial-Uso-y-Gestion-de-Suelo1.pdf

[4] I. Turok and J. Borel-Saladin, "The theory and reality of urban slums: Pathways-out-of-poverty or cul-de-sacs?" *Urban Studies*, vol. 55, no. 4, pp. 767–789, 2018.

[5] N. Bennett, "5. the bottom two billion: the global expansion of urban slums and second-class citizenship," *Contracting Human Rights: Crisis, Accountability, and Opportunity*, p. 54, 2018.

[6] *Streets as Tools for Urban Transformation in Slum*, United Nations Human Settlements Programme, 2012.

[7] L. D'Acci, *The Mathematics of Urban Morphology.* Springer, 2019.

[8] S. Ornes, "Science and culture: Can the principles of topology help improve the world's slums?" *Proceedings of the National Academy of Sciences*, vol. 116, no. 20, pp. 9686–9689, 2019. [Online]. Available: https://www.pnas.org/content/116/20/9686

[9] G. Alexanderson, "About the cover: Euler and königsberg's bridges: A historical view," *Bulletin of the american mathematical society*, vol. 43, no. 4, pp. 567–573, 2006.

[10] L. Bettencourt and G. West, "A unified theory of urban living," *Nature*, vol. 467, no. 7318, p. 912, 2010.

[11] C. J. Stokes, "A theory of slums," *Land economics*, vol. 38, no. 3, pp. 187–197, 1962.

[12] C. Mackliff Cornejo, "Informalidad urbana: Comprendiendo el problema de la tenencia de la tierra en promesa de dios, monte sinai, guayaquil," Master's thesis, PONTIFICIA UNIVERSIDAD CATÓLICA DEL ECUADOR, 2018.

[13] C. V. Mayorga, "Apropiación del espacio en la informalidad: Asentamientos informales en guayaquil," *Territorios en formación*, no. 7, pp. 103–118, 2015.

[14] J. L. Gross and T. W. Tucker, *Topics in Topological Graph Theory.* Cambridge University Press, 2009, vol. 128.

[15] C. P. Bonnington and C. H. Little, *The foundations of topological graph theory.* Springer Science & Business Media, 2012.

[16] J. L. Gross and T. W. Tucker, "Topological graph theory," 1987.

[17] D. S. Richeson, *Euler's gem: the polyhedron formula and the birth of topology.* Springer, 2008, vol. 6.

[18] J. A. Tilley, "The birkhoff diamond as double agent," *arXiv preprint arXiv:1809.02807*, 2018.

[19] M. Stiebitz and B. Toft, "An abstract generalization of a map reduction theorem of birkhoff," *Journal of Combinatorial Theory, Series B*, vol. 65, no. 2, pp. 165–185, 1995.

[20] M. P. Bendsøe and O. Sigmund, "Topology optimization: theory, methods and applications." 2003.

[21] Z. Zuo, "Topology optimization of periodic structures," 2009.

[22] J. Leiva, B. Watson, and I. Kosaka, "Modern structural optimization concepts applied to topology optimization," in *40th Structures, Structural Dynamics, and Materials Conference and Exhibit*, 1999, p. 1388.

[23] X. Huang and M. Xie, *Evolutionary topology optimization of continuum structures: methods and applications*. John Wiley & Sons, 2010.

[24] H. Frank and W. Chou, "Topological optimization of computer networks," *Proceedings of the IEEE*, vol. 60, no. 11, pp. 1385–1397, 1972.

[25] S. Porta, P. Crucitti, and V. Latora, "The network analysis of urban streets," *Environment and Planning B Planning and Design*, 2006.

[26] C. Brelsford, T. Martin, J. Hand, and L. M. Bettencourt, "Supplementary materials for toward cities without slums: Topology and the spatial evolution of neighborhoods," *Science advances*, vol. 4, no. 8, 2018.

[27] A. Rinaldo, R. Rigon, J. R. Banavar, A. Maritan, and I. Rodriguez-Iturbe, "Evolution and selection of river networks," *PNAS*, 2014.

Appendices

A. Code in Python

Here we show the implementation of the main features of the approach. Full code is available on **https://github.com/open-reblock/**.

```
# clean up and probability functions
def WeightedPick(d):
    """picks an item out of the dictionary d, with probability proportional
    the value of that item.  e.g. in {a:1, b:0.6, c:0.4} selects and returns
    "a" 5/10 times, "b" 3/10 times and "c" 2/10 times. """

    r = random.uniform(0, sum(d.values()))
    s = 0.0
    for k, w in d.items():
        s += w
        if r < s:
            return k
    return k


def shorten_path(ptup):
    """ all the paths found in my pathfinding algorithm start at the fake
    road side, and go towards the interior of the parcel.  This method drops
    nodes beginning at the fake road node, until the first and only the
    first node is on a road.  This gets rid of paths that travel along a
    curb before ending."""

    while ptup[1].road is True and len(ptup) > 2:
        ptup.pop(0)
    return ptup


def segment_near_path(myG, segment, pathlist, threshold):
    """returns True if the segment is within (geometric) distance threshold
    of all the segments contained in path is stored as a list of nodes that
    strung together make up a path.
    """
    # assert isinstance(segment, mg.MyEdge)

    # pathlist = ptup_to_mypath(path)

    for p in pathlist:
        sq_distance = segment_distance_sq(p, segment)
        if sq_distance < threshold**2:
            return True

    return False
```

```python

def shortest_path_p2p(myA, p1, p2):
    """finds the shortest path along fencelines from a given interior parcel
    p1 to another parcel p2"""

    __add_fake_edges(myA, p1, roads_only=True)
    __add_fake_edges(myA, p2, roads_only=True)

    path = nx.shortest_path(myA.G, p1.centroid, p2.centroid, "weight")
    length = nx.shortest_path_length(myA.G, p1.centroid, p2.centroid, "weigh

    myA.G.remove_node(p1.centroid)
    myA.G.remove_node(p2.centroid)

    return path[1:-1], length

def find_short_paths(myA, parcel, barriers=True, shortest_only=False):
    """ finds short paths from an interior parcel,
    returns them and stores in parcel.paths  """

    rb = [n for n in parcel.nodes+parcel.edges if n.road]
    if len(rb) > 0:
        raise AssertionError("parcel %s is on a road") % (str(parcel))

    if barriers:
        barrier_edges = [e for e in myA.myedges() if e.barrier]
        if len(barrier_edges) > 0:
            myA.remove_myedges_from(barrier_edges)
        else:
            print("no barriers found. Did you expect them?")
        # myA.plot_roads(title = "myA no barriers")

    interior, road = shortest_path_setup(myA, parcel)

    shortest_path = nx.shortest_path(myA.G, road, interior, "weight")
    if shortest_only is False:
        shortest_path_segments = len(shortest_path)
        shortest_path_distance = path_length(shortest_path[1:-1])
        all_simple = [shorten_path(p[1:-1]) for p in nx.all_simple_paths(myA
                      road, interior, cutoff=shortest_path_segments + 2)]
        paths = dict((tuple(p), path_length(p)) for p in all_simple
                     if path_length(p) < shortest_path_distance*2)
    if shortest_only is True:
        p = shorten_path(shortest_path[1:-1])
        paths = {tuple(p): path_length(p)}

    myA.G.remove_node(road)
    myA.G.remove_node(interior)
    if barriers:
```

```python
        for e in barrier_edges:
            myA.add_edge(e)

    parcel.paths = paths

    return paths

def find_short_paths_all_parcels(myA, flist=None, full_path=None,
                                 barriers=True, quiet=False,
                                 shortest_only=False):
    """ finds the short paths for all parcels, stored in parcel.paths
    default assumes we are calculating from the outside in.  If we submit an
    flist, find the parcels only for those faces, and (for now) recaluclate
    paths for all of those faces.
    """
    all_paths = {}
    counter = 0

    if flist is None:
        flist = myA.interior_parcels

    for parcel in flist:
        # if paths have already been defined for this parcel
        # (full path should exist too)
        if parcel.paths:

            if full_path is None:
                raise AssertionError("comparison path is None "
                                     "but parcel has paths")

            rb = [n for n in parcel.nodes+parcel.edges if n.road]
            if len(rb) > 0:
                raise AssertionError("parcel %s is on a road" % (parcel))

            needs_update = False
            for pathitem in parcel.paths.items():
                    path = pathitem[0]
                    mypath = ptup_to_mypath(myA, path)
                    path_length = pathitem[1]
                    for e in full_path:
                        if segment_near_path(myA, e, mypath, path_length):
                            needs_update = True
                            # this code would be faster if I could break to
                            # next parcel if update turned true.
                            break

            if needs_update is True:
                paths = find_short_paths(myA, parcel, barriers=barriers,
                                         shortest_only=shortest_only)
```

```
                    counter += 1
                    all_paths.update(paths)
                elif needs_update is False:
                    paths = parcel.paths
                    all_paths.update(paths)
            # if paths have not been defined for this parcel
            else:
                paths = find_short_paths(myA, parcel, barriers=barriers,
                                         shortest_only=shortest_only)
                counter += 1
                all_paths.update(paths)
    if quiet is False:
        pass
        # print("Shortest paths found for {} parcels".format(counter))

    return all_paths


def build_path(myG, start, finish):
    ptup = nx.shortest_path(myG.G, start, finish, weight="weight")

    ptup = shorten_path(ptup)
    ptup.reverse()
    ptup = shorten_path(ptup)

    mypath = ptup_to_mypath(myG, ptup)

    for e in mypath:
        myG.add_road_segment(e)

    return ptup, mypath


def choose_path(myG, paths, alpha, strict_greedy=False):

    """ chooses the path segment, choosing paths of shorter
    length more frequently  """

    if strict_greedy is False:
        inv_weight = dict((k, 1.0/(paths[k]**alpha)) for k in paths)
        target_path = WeightedPick(inv_weight)
    if strict_greedy is True:
        target_path = min(paths, key=paths.get)

    mypath = ptup_to_mypath(myG, target_path)

    return target_path, mypath
```

B. Euler's Figures

Although Euler did not use graphs in the original article to treat the problem of the Königsberg's bridges[17], the abstract treatment that he did marked the beginning of the field of graph theory and topology, see Figure 24.

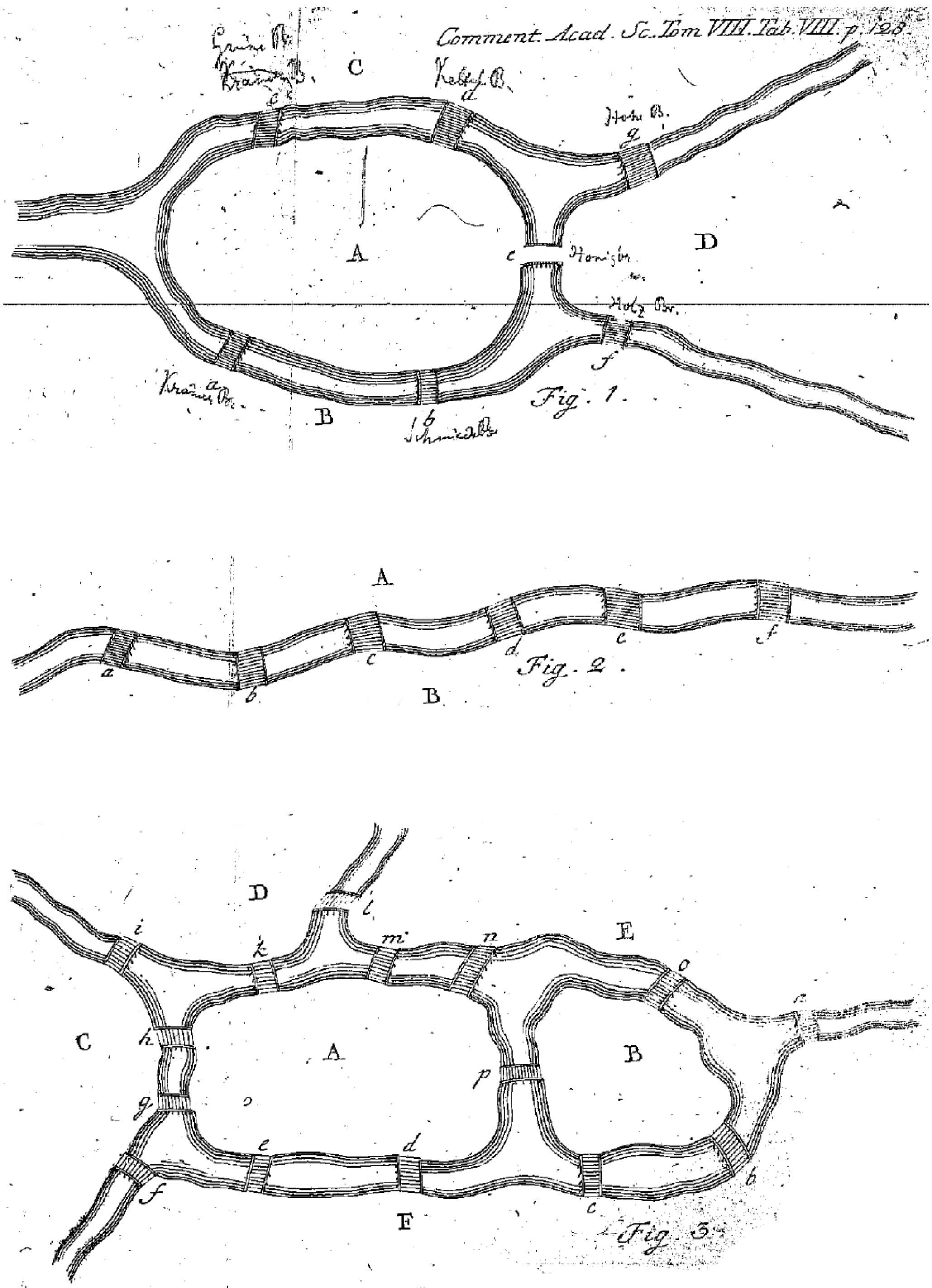

Figure 24: Original figures from Euler's paper. By Euler, Leonhard, "Solutio problematis ad geometriam situs pertinentis", 1741. Euler Archive - All Works. 53. https://scholarlycommons.pacific.edu/euler-works/53